Foundational issues of Chaos and Randomness:
"God or Devil, do we have a Choice?"

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Abstract

This article is about two competing paradigms to explain the nature of our physical world. The prevalent paradigm is 'Randomness', which is intrinsic in Quantum Mechanics (QM) and regarded as essential to our universe, an attribute that led to the famous remark by Einstein – "God does not play dice". While Einstein's comment is regarded by many physicists as indicating an inability of a highly respected scientist to cope with newer developments of physics, the emerging paradigm of 'Chaos' has given credence to Einstein's skepticism.

In order to illustrate the above claim, we develop a hypothetical game in which a visitor arriving on an island has to determine whether the laws of the island are governed by the Devil, who uses 'Randomness', or by an Einsteinian God who uses strict 'Determinism' in the workings of the island. Using Chaos theory, we demonstrate that by any amount of finite measured observations, the observer cannot distinguish between God's universe and Devil's universe. It turns out that Chaos theory is emerging as a very useful tool to analyze natural phenomena and may even develop a more satisfying (and beneficial) alternative explanation than adhoc use of 'Randomness', including possibly an alternative theory to deal with QM and related phenomena. We discuss another important theme in this paper – 'Switching'. The creation of healthy and unhealthy (low-entropy) ferns is obtained through switching between simpler systems. We further discuss switching between deterministic/stochastic systems and by means of counter-intuitive phenomena like Parrando's paradox and Maxwell's demon, we demonstrate the role played by Switching in the creation of Entropy and Complexity. Switching between deterministic systems seems to successfully simulate the counter-intuitive effects created by Randomness.

1. Introduction

The nature of randomness has perplexed the understanding of the human mind since time immemorial. For long, scientists and mathematicians have tried to define randomness, but no satisfying definition has yet been found. What is even more puzzling is the 'unreasonable effectiveness' of randomness as a means of creating complexity and entropy in the universe. However, in this article, we intend to challenge the necessity of randomness for creating structure. We show that deterministic dynamical systems (especially Chaotic ones) can provide an alternate foundation for the creation of complexity and entropy by demonstrating that for any amount of "random" finite measurement data, there always exists a deterministic system producing it. In particular, we argue that the ongoing revolution of Chaos theory is fast advancing and it would not be long when the role of randomness in the foundational aspects of science and mathematics will be seriously challenged.

This paper is organized as follows. In Section 2, we shall discuss about the two important theories of 20th century namely Chaos Theory and Information Theory and their interplay. Specifically, we focus on the unpredictability of deterministic chaotic systems arising out of limitations of computational precision. This paves the way for the thought experiment in Section 3 where we show that any amount of finite observations is insufficient to determine whether the underlying system is deterministic or stochastic. Section 4 deals with switching between systems (stochastic and deterministic) and how it can be yield counter-intuitive phenomena. Importantly, we show that we do not need randomness to arrive at such counter-intuitive results and that switching between two seemingly simple deterministic systems can simulate similar results. The necessity of randomness is seriously questioned. Next, we discuss the two emerging competing foundations of mechanics namely Quantum Mechanics and Chaos Mechanics in Section 5. We only provide a few hints on how Chaos can probably replace Quantum Mechanics. We conclude in Section 6.

2. Deterministic Chaos and Information Theory

Chaos or specifically known as Deterministic Chaos is a phenomenon which although was understood by Pöincare towards the end of the 19th century, it was named only in 1975 [Li 1975] (only in the last three decades of the 20th century.) As the name suggests, it deals with phenomenon that deterministic systems (a system of non-linear ordinary or partial differential equations or discrete maps) give rise to outputs, which are apparently random-like in nature. Let us specifically look at probably the simplest of such systems, known as the Tent map (for e.g., see [Alligood 1997]) as shown in Figure 1.

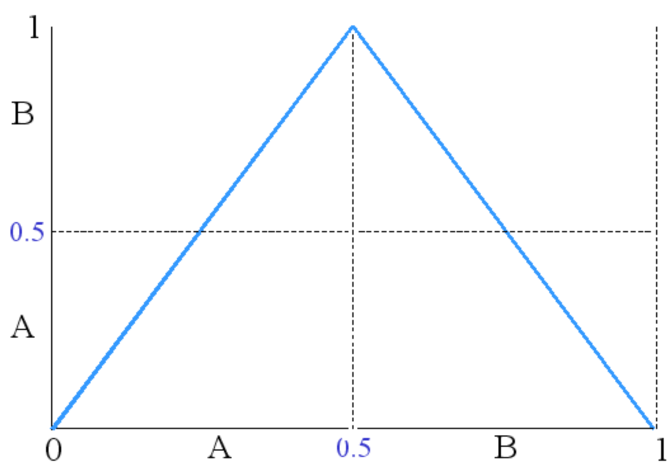


Figure 1. The Tent Map.

$$Y = \begin{cases} 2x & \text{if } 0 \leq x < 0.5 \\ 2-2x & \text{if } 0.5 \leq x \leq 1 \end{cases}$$

Let's take a specific initial condition. For example x_0 = 0.23113851357429. Its first ten iterates are as follows:

0.46227702714858
0.92455405429716

0.15089189140568
0.30178378281136
0.60356756562272
0.79286486875456
0.41427026249088
0.82854052498176
0.34291895003648
0.68583790007295

It is typical to present these iterates as the trajectory generated by the given initial condition. It is well known that if we were to choose a slightly different initial condition, the two trajectories will progressively diverge from each other. This is one of the well-known properties of chaos known as sensitivity to initial condition. This property by itself is also available in linear systems. However the second property of chaos is boundedness. The two together can occur only in non-linear systems and roughly constitute the necessary and sufficient conditions for chaos (for more precise definitions, please refer to [Devaney 1992]).

One more feature that is common with most chaotic systems is that they are "non-integrable". The meaning of this term can be explained by tracing its origin. A solution of a differential equation ($dx/dt = f(x)$) is given by $x = \int f(x)\, dx + c$. Integrability refers to ability to write solutions in closed mathematical form. Most chaotic systems are non-integrable and therefore the route of explicit mathematical solutions is not available. In fact, this is the reason why chaos theory had to wait until the discovery of computers for its proper development. Therefore, in practice, a chaotic equation is solved numerically by means of a computer. This adds fuel to the fire generated by sensitivity to initial condition since at each step of calculation, the computer introduces an inevitable small error owing to its limited precision.

Therefore, on two different computers, even if we have same initial conditions, we may get different answers. This result can be shown in a visually stunning way for the Duffing's oscillator [Alligood 1997]. Physically, this can be seen as a motion of a harmonically driven particle in presence of non-linear restorative forces. For this problem, we begin to take the liberty of assuming that both the position and velocity of a particle are explicitly accurately known and we let the position of the particle evolve. Figure 2 illustrates what happens if this calculation is carried out by two different numerical integration methods.

On the same computer, two highly precise numerical integration routines can give very different answers. There are two consequences of this. One is that there needs to be a more precise definition of what is a chaotic trajectory. Second is that an actual calculation of the chaotic trajectory and its evolution leads to something that from information theory point of view is seen as a generation of entropy. This needs to be examined further. We will begin by stating what we can call as Shannon's paradox.

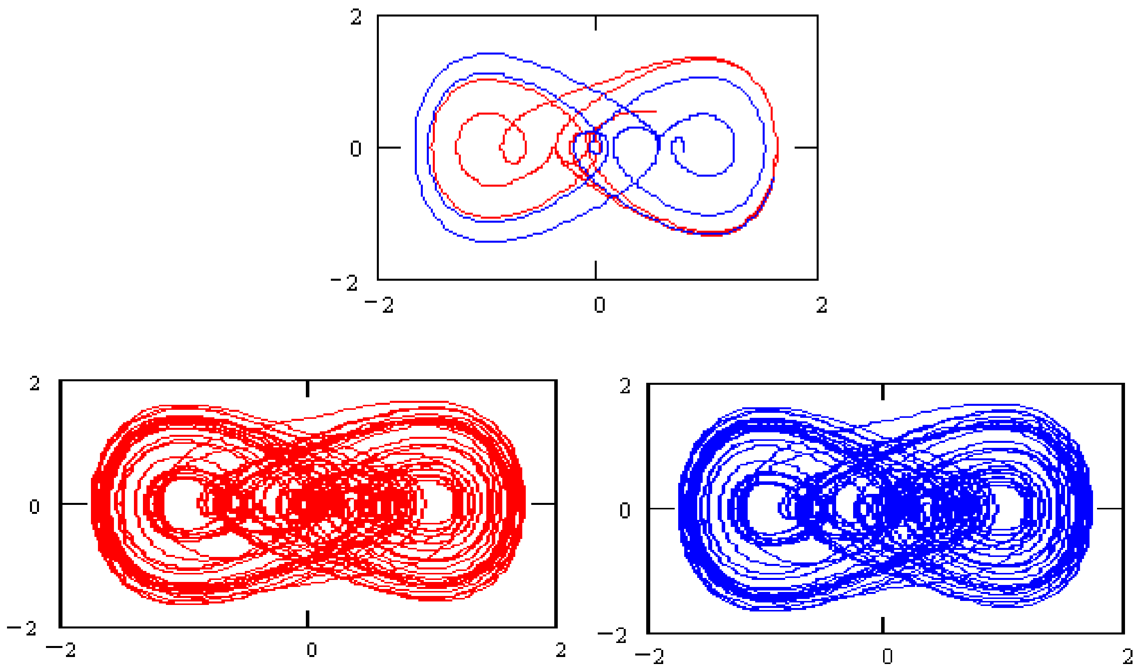


Figure 2. The Duffing Oscillator. **Top:** For the same initial condition, two different numerical integration methods yield very different trajectories (red and blue) in the phase space. **Above:** Pictures are the same. The two trajectories evolve around the same attractor in phase space effectively giving the same picture.

In chaos theory, it is quite common to calculate trajectories as shown above and to calculate the associated Shannon's entropy. Shannon's entropy [Shannon 1948] is defined as the amount of bits needed to represent the information content of the data. It can also be thought of as the amount of uncertainty associated with the data. The theory that deals with entropy and other aspects of information is known as Information Theory. Recently we came across a statement by Shannon that deterministic system can't generate entropy [Kocarev 2001]. This creates essentially a paradox that needs to be solved. Fortunately, we have found a solution that we present below.

First of all, it is quite clear that Shannon's statement is absolutely true if computers were to have infinite precision and if the initial condition was also specified with infinite precision. In that case, even a chaotic system will fail to generate a trajectory with any new information. New information comes in because of finite precision of the computation process. To do this, let us describe the trajectory of our example in alternative form by using the letters A and B. We will say that the trajectory is in A if it belongs to [0, 0.5), in B if it belongs to [0.5,1]. Following this, the above trajectory is represented as "ABAABBABAB". It is clear that some information is lost by replacing the detailed trajectory by these ten letters. However, if we can trust the symbolic sequence, we can pin point to a specific interval in which 'x_0' must reside. In fact, the longer the symbolic sequence, the more precise is the pin-pointing. For details of this calculation, see [Vaidya 2001]. Therefore, the longer the sequence, the lesser the uncertainity of the starting initial condition.

What we propose here is that the above numerical value of the initial condition itself should be seen as a symbolic sequence. Thus 0.23113851357429 represents some number belonging to the interval [0.231138513574290 0.231138513574299]. Given this formulation, now we can see that longer numerical sequences amount to a more and more precise definition of where the initial condition is going to lie. This reduces the possible microstates for the macrostate represented by the observed symbols. This would explain why a development of a chaotic trajectory with the help of a computer is a process that has Shannon's entropy.

The main result of this analysis is as follows. To put chaos theory calculations on a sound theoretical basis, we have to abandon the usual method of a system of differential equations or a system of maps accompanied by specific initial conditions. We have to carry the calculations with the full knowledge that there is going to be inevitable uncertainty about both the true initial conditions and subsequent solution. This in the context of quantum mechanics is one of the immediate contributions of chaos theory because it explains the origin of randomness in a deterministic setting.

3. Who is in Charge? - God or the Devil

In this section, we shall demonstrate that by any amount of finite measured observations, an observer cannot distinguish between Chaos and Randomness, between a deterministic universe and a stochastic universe. We shall perform a thought experiment, involving a game in order to illustrate our point.

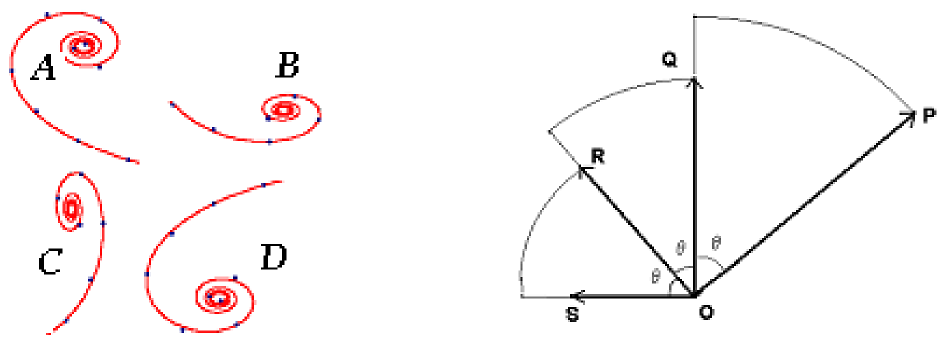
Assume there are two islands, one created and ruled by God, where there is no gambling or randomness and all phenomena are governed by deterministic laws, and another created and ruled by Devil where randomness is allowed. A scientist arrives at one of these islands. The task is that he needs to determine whether God is in charge or the Devil. He is allowed to observe any amount of measurements (but finite).

Rangoli game

Let us assume that the scientist is observing a game being played by 4 players on the island. Each player can press a button to create a Rangoli type of dot from an existing dot. For each player, there is a fulcrum and a fixed rotation and a contraction around that fulcrum. So if a player keeps pressing the button, all we get is a spiral path leading to a fixed point (the fulcrum) as shown in Figure 3. If the players take turns, we end up with a rotation around 4 fixed points.

What happens if at each iteration, the players toss a dice at random and decide who should operate the device for that iteration? The answer is shown in Figure 4 and is completely counter-intuitive at first.

In fact, the "Fern" pattern obtained is a fractal. A fractal is self-similar at various scales. As it can be seen, the same copy of the fern appears as leaves and repeats further among the structure of leaves and so on. One might question, what happens if we do not use randomness fully (we restrict the freedom in some sense). The answer is equally interesting. We get what we call "low entropy ferns", ferns which are not "healthy".



(a) **(b)**

Figure 3. Rangoli Game. (a) Four players A, B, C and D are participating in the game. Each of them has a device, which creates a spiral around a point of their choice. (b) The device used by each of the players to create the spiral. The player begins with a specific origin and direction (OP) and at each iteration rotates with an angle 'θ' shrinking the length by a certain amount to yield the next position (OQ). This is repeated until finally the point converges to 'O'. The point of convergence, rotation angle 'θ' and amount of shrink at each iteration are different for the participants.

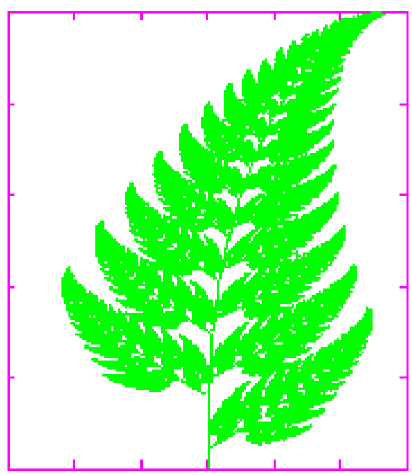


Figure 4. Random switching between players A, B, C and D yields the "Fern" picture above. Such a rich pattern is counter-intuitive. This is a fractal.

It can be seem that the scientist who observes the fern may conclude that it is the Devil's universe since one needs randomness to yield such a rich and beautiful pattern from seemingly simple and boring patterns of the individual systems. But, the scientist would be misled.

In order to see why, we will first start with a simpler problem. Suppose on the Devil's island, the Devil came with a sequence of 0's and 1's using a perfect coin toss. Then he started with some starting value $X=X_0$ (arbitrary) and came out with a sequence by using the equations:

$$X \rightarrow X/2 \quad \text{if} \quad \text{HEADS}$$
$$\rightarrow 1\text{-}\ X/2 \quad \text{if} \quad \text{TAILS}$$

Let the sequence the Devil generated be $\{X_0, X_1, \ldots, X_n\}$. The scientist looks at the sequence and concludes that it was generated by a stochastic or random process and hence it is the Devil who is in charge. The longer the sequence which the scientist records and uses tests for randomness, he finds to his surprise that they are completely uncorrelated and random and the stronger his conclusion.

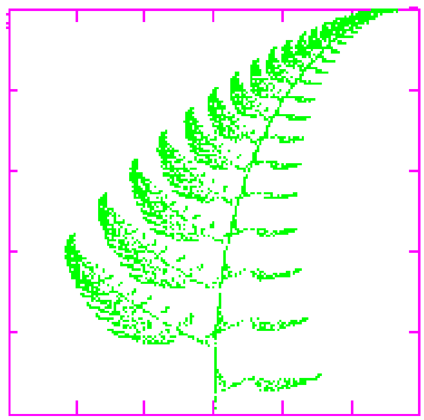


Figure 5. Unhealthy ferns or low entropy ferns are produced if we do not use randomness fully in switching between players A, B, C and D.

Unfolding

When all is nearly over and the scientist has nearly arrived at the conclusion that it is the Devil in charge, it occurs to him that a deterministic system could actually produce the exact sequence, which he has observed. This is due to a method we call unfolding. Consider the following set of equations:

$$Y \rightarrow 2Y \quad \text{if} \quad y \leq ½$$
$$\rightarrow 2\text{-}\ 2Y \quad \text{if} \quad y > ½$$

$$X \rightarrow X/2 \quad \text{if} \quad y \leq ½$$
$$\rightarrow 1\text{-}\ X/2 \quad \text{if} \quad y > ½$$

The above set of equations represents a dynamical system, which is invertible, deterministic and chaotic. Given a sequence of X which is identical to the observed X if as in the example. For any sequence X, we have developed a program to find starting X and Y. This can work for very long sequences provided we have very high precision computer. In any case, we need to show that such a solution exists to cast a doubt on this being Devil's island.

Unfolding Ferns

Unfolding works for ferns also. In fact for any Markov process. What we really need is not randomness but ergodicity to get a full fern. And, chaotic systems can be ergodic.

Ergodicity can be understood as a generalization of the idea of independence. Ergodicity can also be thought of as mixing, in other words, an ergodic process is one where there is good mixing of various trajectories (i.e. the inverse images of an interval which has a non-zero measure eventually spreads and fills the entire space).

On the other hand, there are other properties that chaotic systems can provide which Random systems can't. These are chaotic synchronization, time-delayed synchronization [He 1999] etc. Chaotic synchronization refers to the counter-intuitive phenomena of two chaotic systems starting with totally different initial conditions but eventually converging to the same trajectory when coupled in a specific manner. Time delayed synchronization refers to chaotic synchronization between two chaotic systems with a time delay in the coupling.

4. Switching between Dynamical Systems

In this section, we shall discuss the theme of switching between dynamical systems and how it is an important mechanism to create counter-intuitive phenomenon from simple deterministic systems. We provide a deterministic equivalent of the counter-intuitive phenomenon and question the necessity of randomness in modeling such phenomena.

Parrando's Paradox

Parrando's paradox is a counter-intuitive phenomenon induced by switching between two stochastic processes. The game theoretic analogy of the paradox is that two losing gambling (stochastic) games when alternated yield a winning game (stochastic game) [Harmer 1999]. The converse is also true. The paradox is true for a special class of games. We describe one such example here. These are similar to Astumian's paradox [Astumian 2005] where a winning game is obtained by combining losing games that were modeled on a random walk.

Game A consists of a biased coin with probability of losing ($1/2 + \varepsilon$) slightly higher than the probability of winning ($1/2 - \varepsilon$) where $\varepsilon>0$. Losing implies a loss of Rs. 1.00 and winning a gain of Rs. 1.00. As it can be easily seen, this is a losing game. In other words, if one plays this game a number of trials with an initial capital of Rs. M(0), one quickly starts incurring debt and the average value of the capital decreases with trials (t).

Game B is a little more complicated game. There are two coins B_1 and B_2. If the capital M(t) at t-th trial is a multiple of 3 then coin B_1 is used, else B_2 is used. Coin B_1 is a winning coin where the probability of winning is ($3/4 - \varepsilon$) and probability of losing is ($1/4 + \varepsilon$). Coin B_2 is a losing coin with probability of winning being ($1/10 - \varepsilon$) and probability of losing is ($9/10 + \varepsilon$).). Losing implies a loss of Rs. 1.00 and winning a gain of Rs. 1.00 for both coins. With a little analysis, it can be shown that Game B is a losing game on the average (for $\varepsilon>0$).

The interesting thing happens when you start with an initial capital M(0) and have the option of switching between Games A and B at every trial. We call this as the combined Game C. It turns out that for several different choices of switching the combined Game C is a winning one (for certain range of ε) as shown in Figure 6. Choices of switching for which such a counter-intuitive phenomenon occurs are periodic switching, random switching and deterministic (chaotic) switching. It seems that one might conclude that it is the random or the stochastic nature of the underlying games (A and B) that leads to such a counter-intuitive phenomenon (it is not a paradox in the strict sense as there is no logical contradiction involved here).

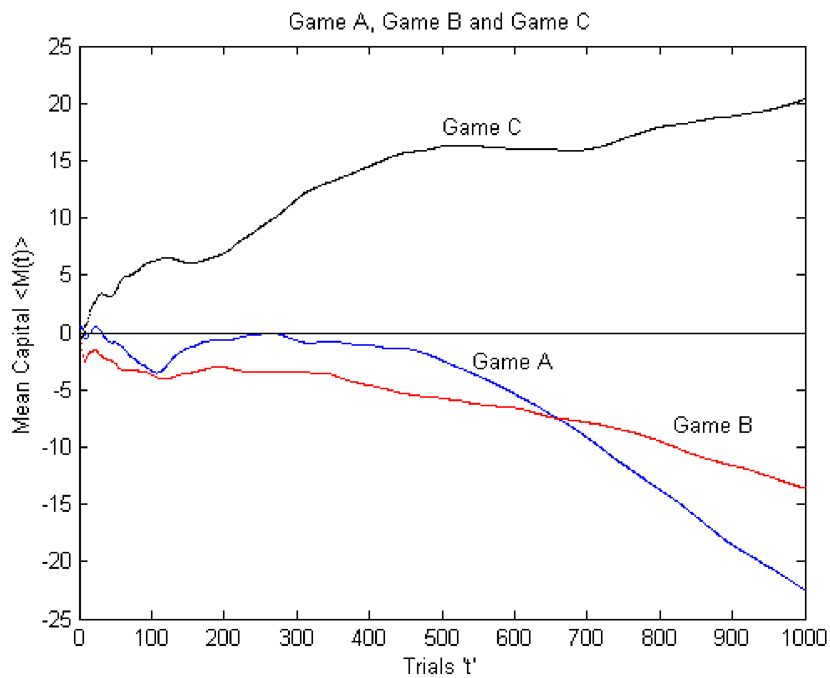


Figure 6. Parrando's Paradox simulated using initial capital $M(0)$ = Rs. 0.00, ε = 0.005. Clearly one can see that Games A and B are losing while Game C which in this case is a random switching between A and B is a winning one.

The explanation of this phenomenon goes back to what are known as Brownian ratchets in thermodynamics [Parrando 2000]. Brownian motion refers to the physical phenomena of minute particles immersed in a fluid moving randomly. A Brownian ratchet, also known as a Flashing ratchet, refers to switching ON and OFF a potential applied to the brownian particle. In the absence of the ratchet, the Brownian particle would diffuse freely in both directions (assume a particle in 1D space) or to make the analogy more accurate, we could add a slight asymmetry to the motion of the particles and they move to the left on an average. When the potential is kept ON continuously, the particles move to the left on an average. However, when the potential is switched randomly between ON and OFF positions, the particle moves to the right on the average. The reason this happens is that the slope of the potential on either directions is asymmetric [Parrando 2000, Parrando 2004].

It turns out that the games described above are a discrete version of the Brownian motion and hence the paradox can be explained. But the question whether the underlying process needs to be a random stochastic process to exhibit such a counter-intuitive phenomena remains. We answer in the negative. We demonstrate a deterministic (not even chaotic) equivalent of the Parrando's Paradox.

Deterministic Parrando's Paradox

We shall show how switching of deterministic dynamics can create Parrando's Paradox.

Consider Game A, which is a deterministic game (no coin tossing, non-random). It consists of first determining the remainder obtained by dividing the current capital M(t) at trial 't' with 3. If the remainder is 0, then a profit of Rs. 2.00 is accrued. If the remainder is 1, then a loss of Rs 2.00 is incurred and if the remainder is 2, then a loss of Rs. 1.00 is incurred. It is easy to verify that for any given choice of initial capital M(0), the game is a losing game. This is also indicated in simulation results shown in Figure 8.

Game B consists of finding two remainders – rem2, which is defined as the remainder of the current capital M(t) when divided by 2 and rem3, which is defined as the remainder of the current capital M(t) when divided by 3. There are two possible values of rem2 (0 and 1) and three possible values for rem3 (0,1,2). There are six possible cases. If rem2=0 and for all the three values of rem3, a loss of Rs. 2.00 is incurred. If rem2=1 and for all the three values of rem3, a gain of Rs. 11.00 is accrued. This is a slightly complicated game, but still completely deterministic and it is easy to analyze this game. It is found to be a losing game.

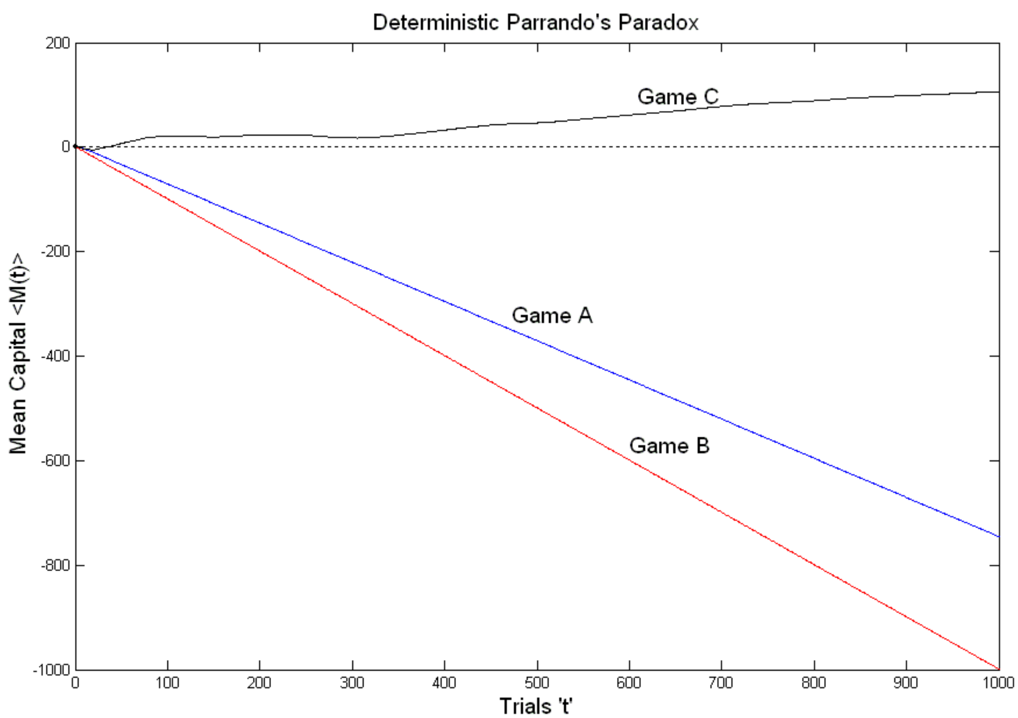


Figure 7. Deterministic (non-Chaotic) Parrando's Paradox simulated using initial capital M(0) = Rs. 0.00. Clearly one can see that Games A and B are losing while Game C which his in this case a random switching between A and B is a winning one. Here the games A and B are deterministic and non-Chaotic.

Game C, as before, is a combined game where the switching between games A and B could be periodic, random or chaotic. It is found that the Game C is a winning game (as shown in Figure 8). The analysis of this game is also quite simple. One can see that both games A and B are like a Markov process (but fully deterministic) and all that matters is what was my previous state (at trial = t-1). Two of the states in Game A are losing states where there is a loss of capital and only one state (remainder = 0) is a profitable state. But notice how once we leave state 0, we never return to it. This is what makes Game A losing. Similarly, Game B has six states of which are three are profitable and three are losing and the transitions are such that once we jump from a profitable state to a losing state, we are always stuck in the losing states (Figure 9). Game C, by employing switching is able to reset this process and increases the frequency of visiting the profitable states in both games. Since the profit is high (Rs. 11.00) every time you visit a winning state in Game B, we see that we make more profit than loss on an average.

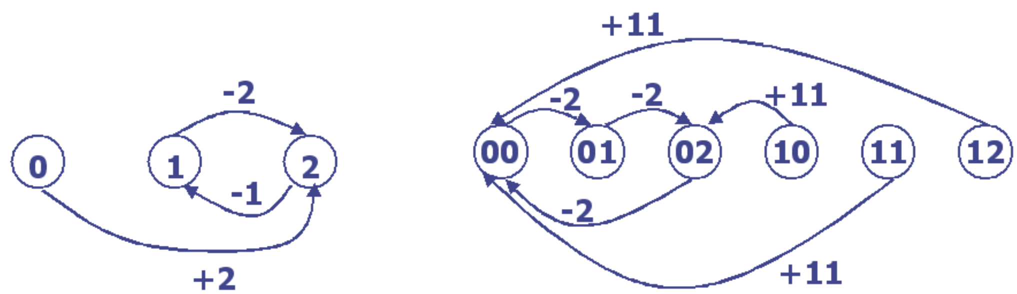


Figure 8. Left: Markov states of remainder for Game A. Right: Game B Markov states. Both games A and B individually are losing. Both games are completely deterministic. Game C which is the combined game is a winning game for periodic, random or chaotic switching.

We have shown how switching between two deterministic systems could yield counter-intuitive outcomes. The above phenomenon can be demonstrated using deterministic chaotic systems also as shown by Chang [Chang 2003]. The Brownian ratchet, which is the inspiration for the Parrando's Paradox could thus be understood as switching between two deterministic systems. In order to do this, the Brownian motion and the ratchet potential need to be modeled as two deterministic dynamical systems. We shall refrain from a further analysis of this, but we hope that we have demonstrated enough motivation by our deterministic Parrando's games that this can be done. What is important to note is that switching between seemingly naïve deterministic dynamical systems could yield a third system who's behaviour is completely counter-intuitive. Rich structure and complex behaviour can be obtained by switching between simple deterministic systems. Randomness per se is not a necessity. There does not seem enough reason to believe that our universe is imposing Randomness even in these highly counter-intuitive phenomena. Therefore, the scientist observing the Parrando's Paradox on the island can not conclude that it is the Devil's universe, it could as well be God's.

Maxwell's Demon

We shall now discuss another intriguing and counter-intuitive phenomenon which goes by the name – 'Maxwell's demon'. We shall argue that Maxwell's demon could also be viewed as a counter-intuitive phenomenon arising out of a simple switching mechanism. The underlying systems themselves may be deterministic or stochastic.

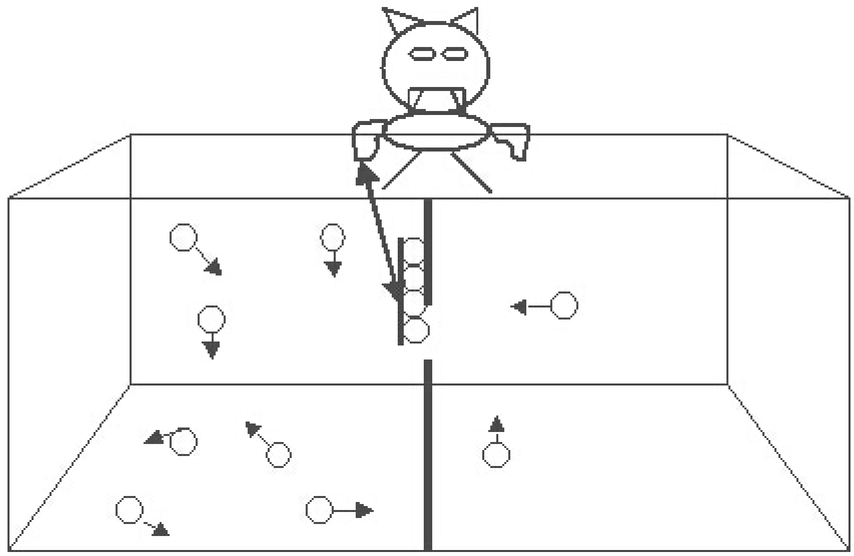


Figure 9. Maxwell's demon. The demon sits on a box which contains freely moving particles. It operates a lid to cover or open a gap in the wall at the center of the box which it opens only for those molecules approaching from the right. Thus after a period of time, most molecules will be found in the left half of the box (as shown above) reducing the entropy of the system and thus seemingly violating the second law of thermodynamics.

The Maxwell's demon tries to defy the second law of thermodynamics. As shown in Figure 9, the demon controls the lid that covers a gap in the wall at the center of the box separating the two halves. To begin with, there are particles spread uniformly in both halves of the box (and equal in number). The demon opens up the lid when it encounters a particle approaching from the right to the left, but closes the lid when it encounters a particle approaching from the left to the right. Over a period of time, there ought to be more molecules in the left half of the box than on the right half. The entropy of the system has definitely reduced (the uncertainty of finding a particle is not the same in the right and the left halves which was the case at the beginning, it has reduced). It is more likely to find a particle on the left half than on the right half. This seems to violate the second law of thermodynamics. The original set-up of the above experiment as proposed by Maxwell is slightly different but the analysis is very similar. We have used the set-up given in [Nielsen 2000].

The way one resolves this apparent paradox is that the system we have shown and described is essentially not isolated. The demon needs to compute the direction and speed of the approaching particles and it needs memory to store this information. If we assume that the memory is finite, it has to perform operations of erasing information and this increases the entropy (as per Landauer's principle). Processing of information also would increase the entropy and the overall entropy of the system (the box + the demon's computing machinery) would necessarily increase with time, thereby increasing the entropy of the total system (which is isolated). The second law of thermodynamics applies only to isolated systems.

The switching metaphor

We can analyze the phenomenon of Maxwell's demon by looking it at from a different perspective. There are two systems here. The first one is the box with the lid fully open and the molecules freely diffusing across the two halves. The second system is the box with the lid fully closed and the molecules freely diffusing in their respective halves (constrained to the half they started with). The demon performs the switching between the two systems to affect an apparent decrease in the entropy of the system. The switching here is not periodic, random or chaotic, but 'intelligent'. It is intelligent in the sense that the switching from the open lid to the closed lid happens precisely when there is a particle from the right approaching the gap (in the correct direction) so that it could easily continue its trajectory into the left half, but no switching happens when a particle approaches the gap from the left. This is a new type of switching and the difference between this switching and that of periodic, random or chaotic switching is in its Kolmogorov complexity, which is a measure of the computational resources needed to specify an object (a string or a program) in some descriptive language (like a programming language such as LISP or encoded on a Turing Machine). The switching rule or a program to perform the switching in the case of periodic, random or chaotic switching needs very few lines of code (in whatever language one might think of, say on a universal Turing machine). This essentially means that the Kolmogorov complexity of such a switching is very low. However, in the Maxwell's demon case, the switching employed needs sufficient 'intelligence' in the sense that the computer program needed to compute the velocities of the particles and to make a decision of whether to switch or not is necessarily going to be quite long leading to a higher Kolmogorov complexity which implies a higher Shannon's entropy. This accounts for the increase in entropy. Could we then say that a reduction in thermodynamic entropy is taken care by an increase in Shannon's entropy?

5. Quantum Mechanics vs. Chaos Mechanics

In this section, we discuss the foundations of physics, specifically we discuss four theories of physics and their inter-relationship. The first two theories are the theory of Relativity and Quantum Mechanics. Both were developed in the early 20th century and posed many serious challenges related to the fundamental postulates of physics. Of these two, we would focus mostly on quantum mechanics. But we do want to mention them together because one of the challenges of the twentieth century would be the unification of these two theories which is not yet completed. Their exists a possibility that the other two theories which are regarded as quite unrelated to these two, may help in this unification. These two theories are Chaos Theory and Information Theory, which we have discussed in Section 2.

Let's enumerate a few of the problems of what led to the acceptance of quantum mechanics. First of these is the use of complex numbers in a way that was more deeply organic than as a matter of intermediate device as it was in electromagnetic theory for example. For a while it appeared, that certain equations of quantum mechanics can't be stated without the use of complex numbers. This is not true as shown by the work of Wigner. The later half of the twentieth century, a view emerged that complex numbers are just a culmination of the process that started with taking the integers, moved on to rationals and to irrationals, each step involved a process of "closure", a notion that has been well formalized in mathematical analysis and algebra. Therefore, the conclusion now is that complex numbers in fact field of N-dimensional complex numbers (even infinite dimensions) should be regarded as natural and something to be expected when we discuss physical phenomenon at any length. Therefore such formulations are here to stay.

The second objection to quantum mechanics has been more serious. It has to do with the issue of randomness. Randomness crept in the epistemological horizon as a mere approximation for certain type of observations, but the quantum mechanical theory makes it a quintessential part related to the quantum mechanical measurement problem. This observation has cast a doubt in many minds about the validity of the quantum mechanical theory itself. Einstein himself summed this skepticism in his famous remark "God does not play dice with the universe." Although people brushed Einstein's remark aside, there is a related quote of Einstein that deserves much attention "Quantum mechanics is an effort to model something that is deterministic and non-linear by linear stochastic means."

Other deeper thinkers in physics have developed elaborate theories like the multi-verse theory [Everett 1957]. These theories maintain that at every instant when there is a measurement, the universe splits in to two and we are aware only one of the two universes. Preposterous as this may seem to some, that it is taken seriously by several eminent scientists shows that the problem of explaining away randomness is something worthy of deeper inspection. Richard Feynman in his classic Lectures in Physics (vol. 3) sums up this problem as follows – "Quantum mechanics is quite different from classical mechanics. In classical mechanics, we firmly believe that a definite experimental set up will have a definite outcome and the role of the science is to develop methods of prediction to find this outcome. In quantum mechanics, alas, such hope is abandoned and definitive predictability has been replaced by description of probabilities of various possible outcomes".

In the previous sections of this paper, we discussed alternatives to randomness based on deterministic chaos theory. Here, we wish to discuss the possibility that Feynman was right about the need to abandon definite predictions, however, this abandonment is required even from a classical perspective. In some ways understanding the origin of the loss of predictability in the classical setting makes this loss intellectually acceptable.

The third problem with quantum mechanics has to deal with issues of entanglement, popularly known in terms of the EPR paradox [Einstein 1935] and the Schrödinger's cat paradox [Schrödinger 1935]. Here again, perhaps the later two theories namely chaos theory and information theory possibly have a serious role to play.

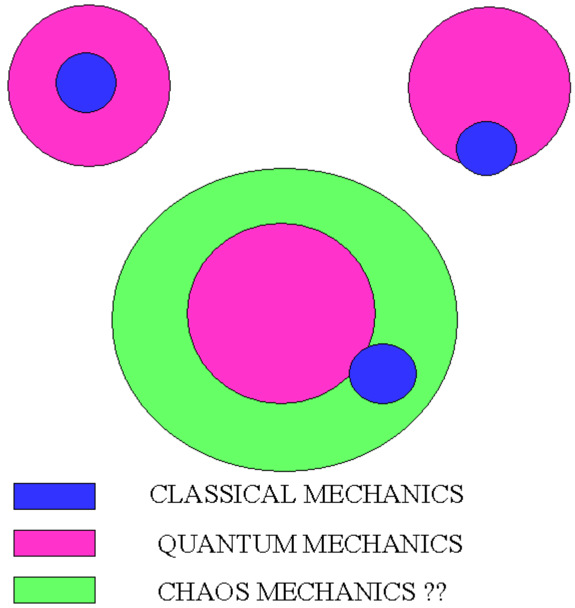


Figure 10. The hypothetical relationship between Classical Mechanics, Quantum Mechanics and Chaos Mechanics. Top left and right diagrams indicate popular opinions. Diagram in the center indicates our view.

We are beginning to see that chaos theory offers some possibilities of dealing with the problems associated with quantum mechanics. But first, let us state what it does not offer. Contrary to what Feynman thought, it does not offer a hope for accurate predictions. The abandonment of the hope for predictions that Feynman lamented because of quantum mechanics and which he thought was not a problem with classical mechanics still persists even in a classical paradigm of chaos. Thus, if chaos is a bridge between classical mechanics and quantum mechanics, this bridge still carries unpredictability with it (Figure 10 shows a hypothetical relationship between the three theories of mechanics). What chaos does seem to offer is a somewhat more aesthetically pleasing genesis of randomness which Einstein would have probably approved of. Because, we are now dealing with non-linear deterministic systems, which are now beginning to show some of the same results that linear stochastic systems [Vaidya 2000]. Specifically, we have chosen some simple Hamiltonians and seen the results of the associated quantum mechanical systems give and come out with chaos based alternative systems which give rise to identical results. How far this process of mimicry goes is yet to be seen. However, some of our preliminary research results show that the phenomenon of chaotic synchronization can be used to model entanglement and may eventually provide an alternate explanation of the famous of EPR paradox.

6. End-note

Whether or not randomness is necessary for the workings of our universe is far from being settled. However, what we can say with certainty is that unpredictability is here to stay. Deterministic chaotic systems are unpredictable unlike the clock-work universe of Newton. Einstein's skepticism in a universe with intrinsic randomness may be after all vindicated. Deterministic chaotic systems are being increasingly used to model various natural phenomena and the underlying theory of dynamical systems is proving to be a powerful replacement to stochastic theory of the universe. We hope we have motivated the reader to re-think the necessity of randomness and shown by means of some counter-intuitive examples, the power of deterministic systems in simulating the effects of randomness. All that we ever have is finite data from our measurements, and for the moment it seems that we can never tell who is in charge – God or Devil.

Dedication

We dedicate this paper in honor of Dr. Raja Ramanna and all the interesting discussions we had with him on this theme.

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