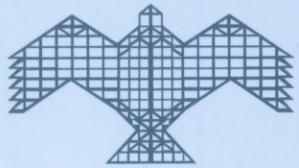

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What Euclid is to Europe, Pāṇini is to India – Or Are They?

FRITS STAAL

NATIONAL INSTITUTE OF ADVANCED STUDIES

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The topic of this evening is the thesis that Euclid is to Europe what Pāṇini is to India.* It is a baffling topic because it causes havoc to a widespread prejudice: the myth of the two cultures of sciences and humanities. The sciences, it is said, are exact and rigorous; the humanities seem to lack those qualities and what they offer instead is not very clear. Since Euclid was a mathematician and therefore a scientist, he belonged to a species that is highly regarded and for good reasons. Pāṇini, on the other hand, was a grammarian or linguist – according to some a more marginal occupation. So must we accept the myth and draw the conclusion, that Europe addressed basic and substantial issues but India dabbled in trivialities?

I hasten to add that I am not attributing any belief in that outlandish myth of the two cultures to academic

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institutions such as the National Institute of Advanced Studies that has honoured me with an invitation. Modern academia includes numerous recent disciplines that transcend such boundaries – I mention only the cognitive sciences and many that deal with information and communication. All the same, popular imagination adheres to the picture and if you want money, especially in the United States, you better claim that what you are doing is science. I am therefore especially grateful and pleased that I am allowed to address the Associates of a National Institute of Advanced *Studies* and not just *Science*s. Perhaps in the final resort, those studies are just sciences since it is, at least in part, a matter of words and definitions.

I am very happy also that I am in a position to visit Bangalore. It is not for the same reason as the prime minister of China, who preceded me in entering India at your cool and leafy city, spurning a red-carpet arrival in New Delhi in order to delve into the heart of India's information technology and globalizing economy. Mr. Wen Jiabao did not come to announce my impending visit and I shall not follow him and proceed to New Delhi, but return to Thailand.

I spent much time in South India but do not know Bangalore well. I missed a rare chance to spend some years here and if I had been able to do it, my Sanskrit would be better. As a graduate student of Advaita Vedanta at the University of Madras, I visited the Shringeri Matha where I was introduced to the then Sankaracarya, Jagadguru Swami

Abhinava Vidya Tirtha. He was quite young himself and was studying a difficult late Advaita text, Vidyāraṇya's *Vivaraṇaprameysaṃgraha*, under the tutelage of Panditaraja V.S. Ramachandra Sastri of the Sankara Matha of your beautiful city. Among the other luminaries, all University Professors, Ramachandra Sastri was the only traditional Pandit member of the first Sanskrit Commission of the Government of India. He did not speak to me but the Jagadguru addressed me in Sanskrit and must have noticed that I did not always get it. Having heard me stutter, His Holiness graciously suggested that I spend some years studying myself with that same great Pandit. But when I went back to the University of Madras, my Professor did not like the idea since I was a Government of India scholar. The Government might not approve such a transition, or approve it soon enough. And so I missed those golden opportunities to learn Sanskrit well and spend some years in Bangalore for which I now must make up fast.

But you came to hear about Euclid and Pāṇini – so where were we? I was holding forth on the difference between the sciences and humanities, arguing that we should not make that distinction. I must now explain briefly how anyone could have come up with such a preposterous idea. It was the pet theory of a nineteenth century German philosopher, Wilhelm Dilthey. Dilthey was basically a historian and did not know much about what he called the natural sciences. His more famous colleague Edmund Husserl, the founder of phenomenology, considered him

chiefly as a man of genius for intuition, not of rigorous science and theory. Dilthey himself preferred to call the sciences of culture *Geisteswissenschaften* or "sciences of the spirit," even though no one knew or agreed on what a *Geist* or Spirit was. Whatever it is, or precisely because of that aura of mystery, the idea about the distinction between sciences of nature and sciences of culture caught on as if with a vengeance.

Then, a century later, Dilthey's ideas were rather unexpectedly supported by the British author and physicist C. P. Snow. It is he who introduced the phrase of "the two cultures". Snow wrote several novels about scientists but that does not mean that he knew anything about the science of literature, let alone the sciences of language, just as an elephant, himself an animal, does not know anything about zoology, let alone the life sciences in general. And so we are left with a distinction between two things, A and B, based upon the ideas of two people, one who knew A but not B, the other B but not A. A shaky foundation.

My conclusion is that we should not make sharp distinctions between large groups of scientific disciplines. Yes, there are many, like pebbles on the beach, but they are not only painted in two colors, red and blue. If you want a substantial foundation on which to base this negative conclusion, you need only glance at the history of science. During the European middle ages, the distinction did not exist. Music, for example, was regarded as a science. If you look at China, India, or the Arab world, there is no trace of

that distinction either. I conclude that we can only study the subject of this evening dispassionately if we accept a very flexible notion of the concept of science. I have found it fruitful to regard as science all forms of systematic knowledge of ourselves and the universe in which we live.

So let us try to turn to Euclid with an open mind. It is not known where he was born, but he was probably taught geometry at Athens by pupils of Plato, and taught and founded a school at Alexandria. His famous *Elements*, which were written around 300 BC, are a systematic treatise on geometry. He may not have been a great mathematician, like some other ancient Greeks, and has been called an excellent schoolmaster. He did indeed write a textbook that incorporated many theorems that others had discovered before him. But I believe that he was more than a schoolmaster because his axiomatic method was new and has so far lasted for almost two and a half millennia. Of course, few things are totally "new." Euclid's text starts with what he calls "common notions", a concept that comes from Aristotle, Plato's star pupil. Common notions are what we now call axioms. Let us take a look at an example: *Things which are equal to the same thing are also equal to each other.* It means that if 3 + 2 equals 5, and 7 – 2 also equals 5, then it follows that 3 + 2 and 7 – 2 are equal to each other. But Euclid was not, or not primarily, thinking of numbers. He was thinking of lines or line segments. His work is first and foremost about geometry though it includes some chapters on numbers. In both areas, his innovation and great contribution was to *prove* or

demonstrate mathematical propositions by deriving them from axioms.

Let us look briefly at one of his propositions. It is not formulated as a statement, as you might expect, but as a construction and says: "To draw a straight line at right angles to a given straight line from a given point on it." The demonstration starts with the sentence: "Let *AB* be the given straight line, and *C* the given point on it." This is accompanied by a figure, which depicts a triangle that stands on the line. In the printed editions of translations of the Elements, these points are marked A, B, C, D, etc. In the medieval manuscripts they are indicated by the letters of the Greek alphabet, *alpha*, *beta*, *gamma*, etc. The proof ends with Q.E.D. or rather the Greek equivalent of the Latin *Quod est demonstrandum,* "which is what had to be demonstrated."

You must not rush to the conclusion that all propositions are constructions. Following Aristotle, Euclid distinguished not only propositions from common notions or axioms. He also introduced other concepts such as hypotheses, postulates and other terms or ideas that he did not always use clearly or even consistently and that specialists continue to discuss. I therefore propose to leave it at that but you need to know one more thing: Euclid paid much attention to definitions. They are actually nicer to quote. The first is: "a point is that which has no part." The second: "A line is breadthless length." And so on it goes.

What were Euclid's shortcomings? There is, of course, a celebrated one: the fifth postulate. The formulation of that postulate by Euclid is quite complex so I shall replace it by a simple approximation. Euclid wrote something like: "Through a point outside a line you can draw only one line that is parallel to the first." That proposition defines flat geometry, it does not apply to the surface of a tennis ball, let alone a mango, and was ousted by the discovery of curved spaces, first in geometry and then in physics and cosmology where they are now used as playthings, more or less. Implicit in the fifth postulate is a deeper and more interesting misconception: the idea that everything in the universe, including ourselves, must be evident to *us*. It is, of course, a preposterous claim. How presumptuous on our part! We as humans are part of the universe. But why should we have been selected in order to understand it, or even evolved in that direction?

I am not quite finished with Euclid. He may have been refuted here or there, but he also demonstrated many things that are simply true. One of them is the so-called theorem of Pythagoras. It is not something that humanity owes to Euclid, or to the Greeks for that matter. It is well known that such theorems are also found in China and India. In India that particular theorem is often, and with equal justification, called the Theorem of Baudhāyana.

Thousands or more such theorems have their place in what is now called modern mathematics and illustrate, in passing, that modern science is not simply a product of the

European Scientific Revolution, but that the way was paved by the earlier history of ancient and medieval science. That history can only be adequately understood if the Eurasian continent is treated as an undivided unit. During that period, the sciences of the Babylonians, Indians, Chinese, Greeks and Arabs were all in contact with each other. It does not follow that Pythagoras, Baudhayāna and the *Chou Pei* borrowed from each other. It would be enough for there to have been borrowings or influences in the area of geometry, one would have led to the other and it is a topic that deserves serious investigation. If, incidentally, you don't believe that that famous and multinational theorem is true, you better give a reason. The best reason is to *prove* that it is false. It will not earn you a Nobel price but you may end up with the Field Medal in mathematics.

I conclude that Euclid's legacy is the axiomatic method which does not belong to Europe but to modern science and therefore to the world.

We are now in a position to make a momentous leap and jump to Isaac Newton, the paragon of modern science, though he regarded himself, like some of his colleagues, as a Natural Philosopher. Newton's *Principia* follows Euclid's method entirely and in all details. He derives propositions from axioms. He accompanies each proposition with a picture and ends with Q.E.D. But his subject matter is different. He adds what were later regarded as physical entities such as *force*. It was a topic about which Aristotle had already speculated, but in the seventeenth century, the

discussion about force was infused with new life by philosophers such as Descartes and Leibniz who were better informed than Aristotle had been. Both these thinkers went beyond Newton in matters of method. The reason is that they made use of the language of *algebra*. Descartes founded what is now called *analytical geometry*, which established a relationship between geometrical figures and algebraic expressions. Leibniz created the largest number of symbolic expressions and notations that are still in use today. As for Newton, he sometimes wrote equations, for example, to express infinite series, but he formulated his most famous laws in Latin which was not always clear. Formulas such as "*f* equals *m* times *a*", which we are now taught in school, were created by later mathematicians such as Euler. It is true that we, today, can easily read them into Newton's Latin expressions, but we do so by hindsight.

Many of the great philosophers and mathematicians, the natural philosophers of the seventeenth century, including Newton and Descartes, preferred the geometry of the ancients, that is, Euclid's *Elements*, to the algebra of the moderns. At the same time, most of them, except Leibniz, regarded that algebra as a barbaric art. The case of Descartes is remarkable and paradoxical because he had himself established a link between geometry and algebra. The dislike of algebra may be due to the fact that it had been introduced by aliens, *paradeśis*, in the first place the Arabs who did not only use what they had inherited from ancient Mesopotamia, India and China, but made

substantial new contributions. The ambivalence about algebraic expressions of the seventeenth century had entirely disappeared a century later, when awareness of the mathematization of physics had reached the point where Lagrange, in his work on analytical mechanics, could declare: "I require neither constructions, nor geometrical arguments, but only algebraic operations."

The case of Newton, the one that concerns us, is the most interesting. He stood on the edge between old and new. Gravity demolished his view of flat space and turned out to be no force at all. It was replaced by a curvature of space-time in which all other particles and forces were subsequently housed. But Newton's view was a simple leftover of Euclid's parallel postulate with its implicit presumption. Newton stood at the end of the geometrical tradition in physics and hesitated to enter the new era of artificial languages. That is an important part of what the economist Maynard Keynes, a lifelong student of Newton, had in mind when he wrote that Newton, who was obsessed by alchemy throughout his life, was the last of the Magi rather than the first modern physicist.

We should not conclude that, because of Newton, Euclid is now done with. There remains his axiomatic method which others applied in different contexts. Spinoza, a kind of Advaitin as you know and another excellent mathematician, applied the art of demonstration from axioms to what he called ethics – a subject that covered a much wider area but included what is nowadays called by

that name. Whether such an approach is fruitful remains to be seen.

Let us retrace our steps. Inspired by Aristotle, Euclid introduced the axiomatic method. He applied it to geometry and Newton extended its use to physics. Its applications to physics and the life sciences are still rather limited, but its applications to logic and mathematics have been spectacular and have led to some of the most celebrated theorems of the last century. One of these is Gödel's incompleteness theorem.

We are now ready to take another, not necessarily greater jump, and go to the second truncated half of the thesis that we are considering, namely: what is Pāṇini to India? Looking at him from the Greek perspective, Pāṇini did something that is tantalizingly similar to Euclid. He composed a grammar of Sanskrit which starts with a list of syllables or sounds, called the *Śivasūtra*, from which all of his grammar can be logically derived.

Since we are in India, I shall be brief about Pāṇini's background. He was, roughly speaking, a contemporary of Euclid. It is a matter on which I expatiated yesterday during the discussion meeting on nature and culture that some of you may have attended. If I had to summarize what I said there and try to adapt it to our context, I would say the following. The background of Pāṇini is the linguistic analysis of the Vedic Prātiśākhya treatises, which started with a discovery that was crucial to the canonization of the Vedas: namely, Śākalya's fixation of the precise form of

the Rigveda. That took place one or two centuries before Pāṇini and was the paradigm that the other three Vedas adopted – the decisive step that led to the canonization of what were subsequently referred to as "The Four Vedas."

Pāṇini, in other words, did not start from scratch. But he was more innovative than Euclid. His grammar incorporated what his predecessors had done, but it revolutionized the science of language. He did not compose separate treatises on the languages of the different schools or *sakhas* of the Vedas like the Prātiśākhyas and from which they derive their name: *prāti-śākhya*, "one for each school." Pāṇini wrote a totally different kind of grammar for the spoken language of his day and thereby laid the foundation for Classical Sanskrit as well as modern linguistics.

If you contrast the discovery of Śākalya or Vidagdha Śākalya, "Clever Śākalya", as he was called, with Pāṇini's *Śivasūtra* you are in a position to appreciate how Pāṇini's grammar turned the science of language upside down. His Vedic predecessors started with KA, KHA, GA, GHA, NA in one direction, and KA, CA, TA, ṬA, PA in another; that is they constructed a *varga* or square. To this they added the vowels and diphthongs: A, AA, I, II, U, UU, R, RR, L, E, O, AI and AU. You do the same in Kannada insofar as I have been able to find out. If I had to summarize what the composers of the Prātiśākhyas contributed to linguistics I would say: they discovered the natural order of the sounds of language – *any* language – as articulated in the mouth

and the surrounding regions, moving from the larynx to the lips, unvoiced to voiced, non-aspirate to aspirate, etc.

To a modern Indian that insight may still be obvious but the Vedic analysis of the sounds of language was a major discovery and a contribution that seems to have been made only once in world history. It was adopted by most of the languages of India and adopted or adapted by many other Asian languages and scripts.

अईउण् । ऋऌक् । एओङ् । ऐऔच् । हयवरट् । लण् ।
a i u Ṅ / ṛ ḷ K / e o N / ai au C / ha ya va ra T / la Ṅ /

ञमङणनम् । झभञ् । घढधष् ।
ña ma na ña na M / jha bha N / gha dha dha Ṣ /

जबगडदश् । खफछठथचटतव् ।
ja ba ga da da Ś / kha pha cha ṭha tha ca ta ta V /

कपय् । शषसर् । हल् ॥
ka pa Y / śa sa sa R / ha L //

The Śivasūtra

Pāṇini was, of course, familiar with these achievements. His *Śivasūtra* starts: a i u Ṇ / ṛ ḷ K / e o Ṅ / ai au C. It is strange, but not yet startling. He continues with semi-vowels and then comes to the consonants I

mentioned just now. They are listed as: ña ma na na na M / (I mark the retroflexes or *mūrdhaṇya* in my Roman transliterations with a dot underneath) jha bha Ñ / gha ḍha dha Ṣ / ja ba ga ḍa da Ś / kha pha cha tha tha ca ta ta V / ka pa Y / and so on. This is rather odd, to say the least.

If you listen to and/or look at these enumerations carefully you will find two strange deviations from what I have called the natural order of the sounds of language. First, Pāṇini garbles or scrambles the order; and second he inserts all kinds of other sounds. The latter can be easily recognized because they are consonants such as Ṇ, K, C, that are *not* followed by a short *a*. Moreover, I have in my recitation stopped after each of these latter sounds to mark a boundary, and the written texts that are before you do the same by making use of vertical bars. I have given this list in Nagari and in Roman, simplifying a little, not only because I cannot do it in Kannada, but because the Roman has one practical advantage over the Indian scripts: it possesses capitals. I have used them to mark the sounds that are *not* followed by a short -*a*- more clearly.

European and American scholars have assumed as a matter of course that Pāṇini's grammar was composed in writing which I consider extremely unlikely. As for the earlier analysis of the Vedic treatises, it was not only about the mouth, but it was certainly done orally, not merely because it belonged to a strictly oral tradition, but because the art of writing had not yet been invented or imported from elsewhere. Pāṇini does refer to writing, that is, he was

familiar with the fact that some people write. He was born, after all, in Salatura, near Attock on the Indus, an area that was then part of the Achaemenid Empire where several kinds of writing had been known for a long time and where an Aramaic script was commonly used for administrative and commercial purposes. Pāṇini was a great scientist and he had, of course, heard of, and may even have been familiar with some shapes or features of that Aramaic script. But he was also a Vedic Brahman and that is one reason for assuming that he composed his grammar orally.

In Europe, Japan, or the United States, I must tell my audiences that the ancient Indians did everything orally which, to them, is astounding; but to you it is nothing new. For me it is, in fact, a stumbling block. The pandits from whom I must learn what I want to know, know everything by heart. They rattle off their Sanskrit at a speed that I do not only fail to comprehend, I can barely follow it. I am sometimes regarded as a specialist in oral traditions but, unfortunately, I myself have to look up everything in books and papers. I am at this point not even referring to my advanced age, at which my memory, which has always been bad, seems at times to have totally given out.

Back to the *Śivasūtra*. It has been called by that name at a much later time, closer to the *bhakti* period and perhaps in order to suggest that it was revealed by the god Śiva. I have retained that attractive appellation, but at Pāṇini's time, and for many centuries thereafter, that initial list was called *Pratyāhārasūtrāṇi* ("The Rules of Condensation")

or *Akṣarasamāmnāya* ("The Enumeration of Syllables.") I must now account for its two strange deviations from what I have called the natural order of the sounds of language. Both can be explained by introducing the concept of *pratyāhāra* or "condensation" that I have just mentioned. Pāṇini *condenses* a set of sounds because he needs them, and only them, for the statement of one of his grammatical rules. That was a momentous innovation and was expressed by means of two other equally great innovations: the use of *metalinguistic markers*, and the use of *metarules* which are *rules* about *rules*, in Sanskrit *paribhāṣā*. The latter concept probably antedates Pāṇini.

I am able to explain these two concepts together, because the consonants followed by a short *-a-* are metalinguistic markers, and their use is explained by the following metarule:

"An initial sound joined to a final metalinguistic marker denotes the intervening sounds as well."

It works as follows. Please look at the picture on page 13. If Pāṇini needs the sounds *a, i* and *u* he simply says *aṆ*. If he needs *i, u, ṛ, ḷ* he says *iK*. If he needs *ya, va, ra, la* he says *yaN*. The two latter condensations are combined to state a rule that explains something with which I am sure many of you are familiar. In Sanskrit, *dadhi* followed by *atra* becomes *dadhyatra*, "milk here", and similarly, *madhu* followed by *atra* becomes *madhvatra,* "honey here." Pāṇini combines these facts with many others and arrives at a general rule which says: "*iK* (that is, *i, u, ṛ, ḷ*) becomes *yaṆ* (that is, *ya, va, ra, la*), respectively, when *aC* (that is,

any vowel) follows."

Paul Kiparsky, who worked with Chomsky at MIT and with S.D. Joshi at Poona, has shown that Pāṇini introduces abbreviatory conventions into his metalanguage, if and only if they make it possible to bring out significant generalizations in the grammar. Economy, in other words, is Pānini's way of achieving generalization. It is a long demonstration and provides a rational explanation for the popular maxim, that grammarians rejoice over the saving of half a syllable as over the birth of a son. It should perhaps be quoted in Sanskrit: *ardhamātrālāghavena putrotsavaṃ manyante vaiyākaraṇāḥ.*

I have a few more things to say about Pāṇini before we shall change course and revert to the original thesis.

First of all, Pāṇini needed, like Euclid and as a matter of course, definitions. Even if they deal with simple things, as some definitions should, they are not simple and sometimes deeply embedded in the grammar. I like the definition of *padam* or "word." It basically says that a word is a nominal or a verbal form, that is, a noun or a verb. Pāṇini defines both these categories in terms of their endings since Sanskrit is, as you know, a language with a rich system of declensions and conjugations. The list of endings of the nominal declensions are condensed and referred to as *suP*. The list of endings of verbal conjugations are condensed and referred to as *tiṄ*. Now comes the definition of "word": "a word is what ends in *suP* or *tiṄ*" or in the original Sanskrit: *suptiṇantaṃ padam.*

Quite as succinct and attractive, I think, as "a point is that which has no part", and, more importantly, an equally significant generalization about our world in which humans and their languages are included. Also another artificial expression that goes beyond Euclid in not being merely stated in a natural language such as Sanskrit or Greek.

Pāṇini's Sanskrit grammar was composed in *sūtra*-form. The genre of *sūtra* or "rule" was destined to have a great future in Sanskrit scientific and philosophic literature. It was later defined as "a brief and concise statement which should capture the essence, be undoubted and face all directions." *Sūtras* are often expressed by a formalized kind of Sanskrit. The Sanskrit of Pāṇini's rules is unintelligible to someone who knows Sanskrit but has not studied Pāṇini. They are highly formalized and resemble formulas. Since they are generally unintelligible in isolation, that is, outside the system in which they played a role, they need to be explained. This led to other new forms of Sanskrit literature, in particular the genre of writing commentaries or *bhāṣya*, subcommentaries, glosses, vrtti, varttika, tika and so on. All these works use expressions and linguistic conventions that are to some extent formalized and have their counterparts in medieval Latin, though Latin is less formalized than the language of Indian linguistics. As for the sutras themselves, Pāṇini's grammar was not only the paradigm, but it remained their most perfect example.

Pāṇini permits himself an occasional joke, or perhaps pleasantry is a better term. One is the final rule of his

grammar which is: *a a*. It is, like many other rules, interrelated with many others and I shall not try to explain it. But it happens to illustrate a recent event in the history of linguistics, a subject on which I shall be even more sketchy than I was in the case of Euclid and Newton.

Pāṇini's grammar led to a vast output of works on grammar, in different schools and with ramifications into other disciplines such as poetics. Grammar was, in India, the Science of the Sciences, and the number of works on grammar may be as large as that of all the other Indian sciences together. One reason is that all writers of Sanskrit studied Pāṇini's grammar or later incarnations such as the grammar of Candragomin that Buddhist authors used. Pāṇini is similar to Euclid also in another respect. Euclid had a great commentator, the Neo-Platonist Proclus who lived seven centuries later. Pāṇini had an even greater commentator, Patañjali who lived two or two and a half centuries later. Perhaps the pace of progress was greater in India than in ancient Greece toward the end of its flourish. Patañjali stated clearly that Sanskrit is infinite, a basic property of language that was implied by Pāṇini's grammar but never stated explicitly since Pāṇini left us nothing but his rules.

Philosophers of language depended on Pāṇini and I should mention at least Bhartṛhari, the greatest and most original of them. Indian linguistics was also influential in other Asian countries, especially in Tibet. In Europe, it led via Franz Bopp to the beginning of modern linguistics.

There is a curious reason for the importance of Pāṇini's grammar to the development of synchronistic linguistics in Europe. That type of linguistics, that treats language not as a development but as a system, was a reaction against the diachronic or historical philology of textual scholars. Its founder, Ferdinand de Saussure, did not like or understand Pāṇini's grammar which is structurally similar to his own. But the reasons, in the case of Pāṇini, were different and based upon an erroneous belief: that of the eternity of Sanskrit. It illustrates that science can grow out of mud just like the lotus which is *paṅkaja*.

Pāṇini's witticism is repeated by the only other linguist who is of a similar stature: Noam Chomsky. Chomsky has never mentioned the name of Pāṇini in any of his numerous writings, as far as I know. But the major work on phonology of the 1960's, a large volume that was written by Chomsky in cooperation with his closest colleague at MIT, the phonologist and Indo-Europeanist Morris Halle, ends with a rule of the same form: $a\ a$. I would not be able to explain it, but it is a small bow to the Indic master that not every reader may appreciate or understand.

Chomsky is well known in India and you may be familiar with his politics, if not his linguistics. When he returned from his first visit to this country, I asked him what did strike him most. He said: the ease of access to the leaders of the government: "I was immediately invited for lunch by Indira Gandhi, but would have to wait very long or for ever before I would be able to see (US) President –",

and here followed a name that I have forgotten. I can imagine one reason. Chomsky is as clever as were Śākalya and Pāṇini. That does not generally hold for an American president.

I conclude that Pāṇini's greatest contribution was the structural system of his grammar which was based upon a unique method and led to deep generalizations, many of which are valid across languages and may be linguistic universals. Insight in the infinity of language was part of that package.

I must now change course and place early Indian grammar in the scientific context in which it belongs and fits most closely: that of the science of ritual or *kalpa*. It is the first in the Vedic list of sciences and includes geometry. You may wonder whether there exists such a science and where it comes from. It is a large topic and I don't have much time but shall treat one example: the notion of *default*. It is, like the concept of metarule, a feature of both the science of ritual and that of language. The two sciences share many structural features. In many cases, the ritual analysis seems to have preceded that of the grammarians and that may apply to the discovery of default options also.

The clearest account of the notion of default occurs in the Śrauta Sūtra of Āpastamba. The Āpastambas lived on the banks of the Yamuna river in the region of Mathura during the Kuru period, but their Śrauta Sūtra was composed further east and later, perhaps as late as Pāṇini. It singles out the default options for oblations, priests and

ritual implements. It specifies that the default oblation is clarified butter (*juhotīti codyamāne sarpirājyam pratiyāt*; I shall continue to quote them in Sanskrit so that you can appreciate how brief most metarules of ritual are). The metarule on the default oblation means that if an oblation is prescribed, but it is not specified what it is an oblation of, it has to be assumed that it is an oblation of clarified butter. If it is an oblation of anything else, it will be stated explicitly. The default priest is the Adhvaryu (*adhvaryuṃ kartāram*). That means that if a *sūtra* says that "he" has to perform such and such an act, the "he" refers to the Adhvaryu. If another priest has to perform an act, the name of his office will be specified. Among the ritual implements, the default implement is the *juhū* ladle (*juhūm pātram*). There are degrees of default: when the *juhu* is already used, and no other implement is specified, the oblation has to be made with the help of the *sruva* (*vyaprtayam sruvena).*

The notion of multiple default echoes or is echoed by Pāṇini's metarule: *anabhihite*, "(the following rules apply) when it [i.e., the *kāraka*] is not (already) expressed." It is a very abstract metarule that appears in the section on syntax in which *kāraka* relations are studied. These are relations that are similar to the relations between subject and object, subject and indirect object, and the like. It is a rather technical topic in any grammar, and certainly in that of Pāṇini. There exists an extensive literature on *kāraka* in the Sanskrit tradition, in modern Indian languages such as Hindi and Marathi, in English, French, German and

Japanese. I shall not attempt to explain any of it. All it shows is that the notion of default is an early notion that seems to have traveled from the science of ritual to that of language. And there is a technical term for it: *anabhidhānam.*

Young Bangaloreans may be inclined to believe that the concept of default originated in Bangalore. They are partly right, since it happened in India, not in Karnataka but in Kosala or Videha, now eastern Uttar Pradesh and Bihar north of the Ganges. That is, it happened at the eastern extremity of Vedic India and at the end of the Vedic period.

Back to our thesis: "What Euclid is to Europe, Pāṇini is to India", but we must now address the question I added in the title of my talk: "Or Are They?"

To begin at the end – we must, in the case of India, add the science of ritual to the science of language. It is not redressing the balance, but rather the opposite: add qualifications to what seemed to be a balanced comparison between two sciences in two civilizations. It is really two in one and one in the other.

We have seen that Pāṇini was preceded by clever Śākalya in the discipline of grammar. If the science of ritual is equally important, we are entitled to know the name of an equally clever ritualist. It is easy, for it can only be Baudhāyana, who was far more important than Āpastamba, since he was the author of the most detailed and authoritative of the ritual sutras. His name is the same as that of the author of the geometrical theorem that is also

called after Pythagoras, for both Baudhāyanas belonged to the same Vedic school of that name and their apparently personal names reflect that affiliation.

These additions amplify and complicate the symmetry of the original thesis, that Euclid is to Europe what Pāṇini is to India. But there are other asymmetries in store.

One is concerned with the position and status of the sciences we have considered within their respective traditions. The science of grammar or linguistics that was put on a firm foundation by Pāṇini seems to constitute a unique event in the history of mankind. No other civilization has created anything similar until we come to modern linguistics, which was not only influenced by the Indian tradition, but would not exist without it. We can not say the same about the geometry of Euclid, but it does apply to his axiomatic method. So here we have a unique science on the one side, and a unique method on the other. An interesting but asymmetrical relationship.

This particular asymmetry is related to another: while the mathematician Euclid in Europe was not accompanied or followed by numerous systems of linguistics, the linguist Pāṇini lived in a country that can boast of a great tradition of mathematicians. Apart from the Baudhāyanas and their Vedic colleagues, I need only mention Āryabhaṭa and Brahmagupta in addition to the extraordinary geniuses of Mādhava in the fourteenth and Ramanujan in the twentieth century. But India also made the anonymous contribution of the Indian numerals and the zero. These appeared

elsewhere in the world in some form or other, but are in their Indian form an essential part of civilization. If the natural order of the sounds of language, that was discovered in India and adopted and adapted in many writing systems of Asia, had also reached the Near East and Europe, there would not be so many irrational and messy alphabets like the *ABC*, and the modern world would have rational and practical Indian syllabaries as well as rational and practical Indian numerals.

We would like our life and our sciences to be simple but it did not happen for reality is often more messy than it appeared to be at first. There may be more to it, but with these imbalances I have explained the *Or Are They* of my title and take my leave.

Bibliography

Agrawala, V.S. (1953), *India as known to Pāṇini.* Lucknow: University of Lucknow.

Boehtlingk, Otto (1887, 1964), *Pāṇini's Grammatik.* Hildesheim: Georg Olms Verlagbuchhandlung.

Bronkhorst, Johannes (2001), " Pāṇini and Euclid: Reflections on Indian Geometry," *Journal of Indian Philosophy* 29: 43-80.

Caland, W. (1903, 1966), *Über das rituelle Sūtra des Baudhāyana.* Leipzig: Brockhaus.

Cardona, George (1969), "Studies in Indian Grammarians I: The Method of Description Reflected in the Śiva-sūtras," *Transactions of the American Philosophical Society* 59/1:3-48.

-- (1980), *Pāṇini. A Survey of Research.* Delhi-Varanasi-Patna: Motilal Banarssidas.

Chatterji, Kshitish Chandra, ed. and transl., *Paspaśā to Patañjali's Mahābhāṣya.* Calcutta: A. Mukherjee & Co.

Datta, Bibhutibhusan and Singh, Avadesh Narayanan (1935-1938), *A History of Hindu Mathematics: A Sourcebook,* I-II. Lahore: Motilal Banarsi Das. Reprinted 1962 Bombay: Asia Publishing House and 2001: Delhi: Bharatiya Kala Prakashan.

Deshpande, Madhav M. (1997), "Who inspired Pāṇini"? *Journal of the American Oriental Society* 117/3 pp.444-65.

Dijksterhuis, E.J. (1986), *The Mechanization of the World Picture. Pythagoras to Newton.* Princeton: Princeton University Press.

Dilthey, Wilhelm (1949), *Grundriss der allgemeinen Geschichte der Philosophie.* Frankfurt am Main.

Faddegon, Barend (1926), "The Catalogue of Sciences in the Chāndogya-Upaniṣad," *Acta Orientalia* 4: 42-54.

-- (1929), "The Mnemotechnics of Pāṇini Grammar," *Acta Orientalia* 7: 48-65.

Flood, Gavin, ed. (2003), *The Blackwell Companion to Hinduism.* Blackwell Publishing.

Griffiths, Arlo and Jan E. M. Houben, eds. (2004), "The Vedas. Texts, Language & Ritual", *Proceedings of the Third International Vedic Workshop,* Leiden 2002, Groningen: Egbert Forsten.

Hayashi, Takao (2003), "Indian Mathematics," in: Flood, ed., 360-75.

-- (2000), "Indian Mathematics," *The History of Mathematics from Antiquity to the Present: A Selective Annotated Bibliography,* ed. Joseph W. Dauben, 1985, New York:

Garland. Revised edition on CD-ROM ed. Albert C. Lewis in cooperation with the International Commission on the History of Mathematics. Providence: American Mathematical Society.

Heath, Thomas L. (1956, many reprints), *The Thirteen Books of Euclid's Elements.* Cambridge: Cambridge University Press. New York: Dover Publications.

Hinüber, Otto von (1989), *Der Beginn der Schrift und frühe Schriftlichkeit in Indien.* Stuttgart: Franz Steiner. Akademie der Wissenschaften und der Literatur, Mainz.

Ingalls, Daniel H.H. (1954), "The Comparison of Indian and Western Philosophy," *Journal of Oriental Research* 22: 1-11.

Jha, V.N. (1976), "Stages in the composition of the Ṛgveda-Padapāṭha," *Bulletin of the Deccan Cllege Research Institute* 25/3-4: 47-50.

Joshi, S.D. and S.D. Laddu (1983), *Proceedings of the International Seminar on Studies in the Aṣṭādhyāyī of Pāṇini.* Pune: University of Poona.

Keynes, John Maynard (1951),"Newton, the Man," in: *Essays in Biography,* London: Rupert Hart-Davis, 310-323.

Kielhorn, F., ed. (1960-62), *The Paribhāṣenduśekhara of Nāgojībhaṭṭa.* Poona: Bhandarkar Oriental Research Institute.

Kiparsky, Paul (1979), *Pāṇini as a Variationist.* Cambridge: MIT Press and Poona: Poona University Press.

-- (1982), *Some Theoretical Problems in Pāṇini's Grammar.* Poona: Bhandarkar Oriental Research Institute.

-- (1991), "Economy and the Construction of the Śivasūtras." In: Madhav M. Deshpande and Saroja Bhate, ed., *Pāṇinian Studies. Professor S. D. Joshi's Felicitation Volume.* University of Michigan: Center for South and Southeast Asian Studies, pp. 239-61.

-- (2002), *On the Architecture of Pāṇini's Grammar.* Stanford: Centre for Linguistics.

Mahadevan, T.M.P. (1957), *The Philosophy of Advaita with special reference to Bhāratītīrtha-Vidyāraṇya.* Madras: Ganesh & Co.

Mates, Benson (1986), *The Philosophy of Leibniz. Metaphysics and Language.* New York – Oxford: Oxford University Press.

Netz, Reviel (1999), *The Shaping of Deduction in Greek Mathematics. A Study in Cognititve History.* Cambridge etc.: Cambridge University Press.

Neugebauer, Otto (1957), *The Exact Sciences in Antiquity.* Providence: Brown University Press.

Newton, Isaac (1729, 1934, 1962 etc.), *Newton's Principia,* Motte's Translation Revised by Cajori. Berkeley etc.: University of California Press.

Park, David (1988), *The How and the Why. An Essay on the Origins and Development of Physical Theory.* Princeton: Princeton University Press.

Pingree, David (1981), *Jyotihśāstra. Astral and Mathematical Literature.* Wiesbaden: Otto Harrassowitz.

Renou, Louis, (1960), "La forme et l'arrangement interne des Prātiśākhya," *Journal asiatique* 1-40.

-- (1941-42), "Les connexions entre le rituel et la grammaire," *Journal asiatique* 233: 105-65. Reprinted in Staal 1972: 434-469.

-- (1963), "Sur le genre du sūtra dans la literature sanskrite," *Journal asiatique* 165-216.

Ruegg, David Seyfort (1978), "Mathematical and Linguistic Models in Indian Thought. The case of Śūnyatā," *Wiener Zeitschreift fur die Kunde Sudasiens* 22: 171-181.

Scharfe, Hartmut (1961), *Die Logik im Mahābhāṣya.* Berlin: Akademie-Verlag.
-- (1971), *Pāṇini's Metalanguage.* Memoirs of the American Philosophical Society, Philadelphia.
Snow, C.P. (1959), *The Two Cultures and the Scientific Revolution.* Harmondsworth: Penguin Books.
-- (1963), *The Two Cultures: A Second Look.* Harmondsworth: Penguin Books.
Staal, Frits (1962), "A Method of Linguistic Description: The Order of Consonants according to Pāṇini," *Language* 46: 502-507.
-- (1963), *Euclides and Pāṇini. Twe methodische richtlijnen voor de filosofie.* Inaugural Lecture, University of Amsterdam. Amsterdam: Polak en van Gennep.
-- (1963), "Sanskrit and Sankritization," *Journal of Asian Studies* 22: 261-75. Reprinted in *South and Southeast Asia: Enduring Scholarship,* ed. J. A. Harrison, Tucson, 213-17.
-- (1965), "Euclid and Pāṇini," *Philosophy East and West.* 15: 99-116.
-- ed. (1972), *A Reader on the Sanskrit Grammarians.* Cambridge & London: The MIT Press. Reprint (1985), Delhi: Matilal Banarsidass.
-- (1982), *The Science of Ritual.* Poona: Bhandarkar Oriental Research Institute.
-- (1982), "Ritual, Grammar and the origins of Science in India," *Journal of Indian Philosophy* 10: 3-35.
-- (1988), *Universals. Studies in Indian Logic and Linguistics.* Chicago and London: The University of Chicago Press.
-- (1989), "The Independence of Rationality from Literacy," *European Journal of Sociology* 30: 301-10.
-- (1989, 1993), *Rules without Meaning: Ritual, Mantras and the Human Sciences.* New York etc.: Peter Lang. Reprint 1996: *Ritual and Mantras: Rules without Meaning.* Delhi: Motilal Banarsidass.
-- (1992), "Sūtra," in: Vatsyayan: 303-14.
-- (1993), *Concept of Science in Europe and Asia.* Leiden: International Institute for Asian Studies.
-- (1995), "The Sanskrit of Science," *Journal of Indian Philosophy* 23: 73-73-127
-- (1999), "Greek and Vedic Geometry," *Journal of Indian Philosophy* 27: 105-127.
-- (2001), "Squares and Oblongs in the Veda," *Journal of Indian Philosophy* 29: 258-273.
-- (2001), "La Scienza nella Cultura Indiana," in: Torella, ed.:611-638.

-- (2002) "There is No Proof But...Addendum to EJVS 7-3 on Vedic Geometry and the History of Science," *Electronic Journal of Vedic Studies (EJVS)* 7-5.
-- (2003), "The Science of Language," in: Flood, ed., 348-59.
-- (2004), "From *prānmukham* to *sarvatomukham:* A Thread through the Śrauta Maze," in: Griffiths and Houben, eds., 521-55.
-- (2005), "Artificial Languages across Sciences and Civilizations," *Journal of Indian Philosophy*
-- *(forthcoming), KA KHA GA of the Vedas.*
-- *(forthcoming),* "The Sound Pattern of Sanskrit in Asia," *Sanskrit Studies Centre Journal* (Silpakorn University, Bangkok).
Thieme, Paul. (1931), "Grammatik und Sprache, ein Problem der altinidischen Sprachwissenschaft," *Zeitschrift for Indologie und Iranistik* 8:23-32.
-- (1935), *Pāṇini and the Veda. Studies* in *the Early History of Linguistic Science* in *India.* Allahabad: Globe Press.
Torella, Raffaele, ed. (2001), *La Scienza Indiana* in: *Storia della Scienza:* 609-949.
Truesdell, C. (1968), *Essays* in *the History of Mechanics,* New York: Springer.
Vatsyayan, Kapila, ed. (1992), *Kalātattvakośa. A Lexicon of Fundamental Concepts* in *the Indian Arts.* New Delhi: Indira Gandhi National Centre for the Arts and Delhi: Motilal Banarsidass Publishers.
Van der Waerden, B.L. (1983), *Geometry and Algebra in Ancient Civilizations.* Berlin etc.: Springer Verlag.
Verhagen, P.C., (1994), *A History of Sanskrit Grammatical Literature in Tibet,* Vol.I: *Transmission of the Canonical Literature.* Leiden etc.: E.J. Brill.
-- (1994), *A History of Sanskrit Grammatical Literature in Tibet,* Vol.II: *Assimilation into Indigenous Scholarship.* Leiden etc.: E.J. Brill.
Whiteside, Wlliam D. (1969), *The Mathematical Papers of Isaac Newton.* Vol.III. Cambridge: The University Press.

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