

REVIEW

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# Harnessing hyperentanglement and hybrid entanglement for sustainable quantum batteries and environmental sensing

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## Abstract

Quantum hyperentanglement and hybrid entanglement are advanced quantum resources with significant potential for technological innovation. This review investigates the proposed role of these entanglement structures in the optimization of two critical quantum technologies: quantum batteries and quantum sensors. We synthesize existing literature to analyze how hyperentangled states could enhance energy storage systems by increasing ergotropy density, accelerating charging rates, and enabling parallelized energy transfer. Furthermore, we examine how hyperentangled and hybrid-entangled architectures may improve the sensitivity, dynamic range, and robustness of quantum sensors through multi-parameter estimation and decoherence suppression. By contextualizing these advancements within quantum thermodynamics and metrology, we identify key theoretical advantages, discuss the significant experimental challenges that must be overcome for their realization, and propose a roadmap for future research. Our findings suggest that high-dimensional entanglement engineering could serve as a foundational principle for designing next-generation quantum devices aimed at addressing global sustainability challenges.

**Keywords** Quantum computing, Quantum sensing, Battery energy storage system

## 1 Introduction

Quantum engineering has contributed towards sustainability, such as with quantum photovoltaics, quantum materials for energy storage, quantum magnetometry for non-invasive exploration of natural resources, quantum chemistry for sustainable materials and quantum computing for optimization problems in areas like smart grid efficiency enhancement [1–11]. At the heart of many quantum technologies is the principle of quantum entanglement - a form of non-classical, weakly non-local correlation between the quantum states of two or more particles, which seems to defy classical locality [12, 13]. While not all sustainable technologies rely on entanglement, its unique properties can be leveraged to enhance certain applications within the realm of sustainability. For example, multipartite entanglement is an important resource for achieving quantum



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advantage in sensing [13, 14]. This capability can help with effective environmental management and conservation efforts. The pursuit of optimal quantum battery architectures has been a vibrant area of research, providing a crucial comparative foundation for this review. For instance, studies have systematically compared charging protocols, revealing the distinct advantages of parallel over series configurations in specific qubit networks [15, 16]. Furthermore, the dynamics of energy exchange have been shown to be fundamentally dependent on entanglement between a charger and a battery, with the ergotropy directly linked to the degree of bipartite entanglement [17, 18]. Other work has explored optimizing performance via multi-photon transitions, demonstrating control over energy storage, power, and operational stability [19]. The influence of environmental interactions and qubit-cavity coupling strengths has also been rigorously analyzed, providing benchmarks for coherence and power output. This review builds upon these established optimization strategies by exploring how hyperentanglement and hybrid entanglement—resources that operate across multiple degrees of freedom—could offer a new pathway to surpass the performance metrics identified in these prior works.

In hyperentanglement and hybrid-entanglement, we have a more complex form of entanglement where particles are entangled within and across multiple degrees of freedom [20]. Hyperentanglement presents unique opportunities to further advance sustainability through innovations in quantum batteries and sensors. Quantum batteries utilize the principles of quantum mechanics to achieve rapid charge transfer and high energy density [21]. We show that by integrating hyperentanglement and hybrid-entanglement into their design, these batteries could enhance energy storage efficiency and reduce the environmental impact associated with conventional energy systems. Similarly, hyperentangled and hybrid entangled states can significantly elevate the performance of quantum sensors used in environmental monitoring. These sensors already offer exceptional sensitivity and precision; however, incorporating entanglement within and across multiple degrees-of-freedom can further improve their capabilities, enabling multi-modal real-time data encoding that can help with addressing climate change and other pressing environmental issues. As we confront urgent global environmental challenges, the advancements discussed in this paper illuminate the potential of quantum technologies to foster a sustainable future through enhanced energy solutions and improved environmental monitoring capabilities. Through this exploration, we aim to contribute to the ongoing dialogue on integrating cutting-edge quantum engineering with sustainability initiatives. This paper is structured as a forward-looking review. Section II provides an overview of quantum battery principles before analyzing the potential role of hyperentanglement. Section III does the same for quantum sensing. Section IV presents a unified discussion of the opportunities and challenges, including experimental feasibility, and Section V concludes with a research roadmap

## 2 Quantum battery

The quantum analogue of the Carnot engine can operate between reservoirs at temperatures of opposite sign [22]. However, entanglement cannot be used to get more work from an energy reservoir [23]. Thermodynamic at equilibrium does not impose constraints on the rate at which energy is transferred to work or heat, and it has been seen that, for many-body quantum systems, generation of entanglement can be associated with faster extraction of work [23–25]. Quantum batteries are temporary systems for

storing energy, with an associated finiteness in power density and energetic capacity, that lose energy to the environment [26–28]. The Dicke quantum battery was the first solid state quantum battery devised, followed soon by SYK fermionic batteries and spin-chain quantum batteries [29–31]. Nonclassical correlations have been found to enhance charging power at a rate faster than the number of sub-cells [27, 32, 33].

## 2.1 Fundamentals of quantum batteries

We can define a quantum battery in terms of a  $d$ -dimensional system whose internal Hamiltonian  $H_0 = \sum_{k=1}^d \epsilon_k |k\rangle\langle k|$  has partially degenerate energy levels  $\epsilon_k \leq \epsilon_{k+1}$  and non-zero bandwidth  $w[H_0] = \epsilon_{\max} - \epsilon_{\min} > 0$ , where  $\epsilon_{\max}$  and  $\epsilon_{\min}$  are the largest and smallest eigenvalues respectively. We can charge or discharge a quantum battery using a cyclic unitary process described by  $\dot{\rho} = -i[H_0 + H_1(t), \rho(t)]$ , where  $H_1(t)$  is a Hermitian time-dependent interaction that is turned on at  $t = 0$  and off at time  $t = \tau$ , with  $\hbar$  being unity. We can consider the energy deposited in terms of

$$W(\tau) = \text{Tr}[H_0 U(\tau; 0) \rho_0 U^\dagger(\tau; 0)] - \text{Tr}[H_0 \rho_0]$$

where  $U(\tau; 0) = \mathcal{T}\{-i \int_0^\tau ds [H_0 + H_1(s)]\}$  with the time-ordering operator  $\mathcal{T}$ . We can then define characteristics such as ergotropy  $\varepsilon$  as the maximal amount of work that can be extracted from a state  $\rho$  via unitary operations:

$$\varepsilon(\rho) = \text{Tr}[H_0 \rho] - \min_{U \in SU(d)} \text{Tr}[H_0 U \rho U^\dagger]$$

If cannot extract any work from a specific state, then that state is known as passive, which happens to be the case when it is diagonal in the basis of the Hamiltonian  $H_0$ , and its eigenvalues do not increase with the energy [34–36]. There has been some discussion on how entanglement will affect ergotropy: generation of entanglement among multiple subsystems may have extract ergotropy but more recent works have shown that separability of subsystems can be maintained for harvesting ergotropy if two-body operations are available [23, 26]. For a charging operation, we can define the charging power as

$$P(t) = \text{Tr}[H_0 \dot{\rho}(t)]$$

defined in terms of an energy-gain with respect to the internal Hamiltonian  $H_0$  of the battery. Then we can define the average power as

$$\langle P \rangle_\tau = \frac{W(\tau)}{\tau}$$

Entanglement is key to effectively maximizing both the injected work and the charging power.

## 2.2 Hyperentanglement for enhanced energy storage

In the context of quantum thermodynamic devices, hyperentanglement—the simultaneous entanglement of quantum systems across multiple, orthogonal degrees of freedom (DoFs), such as spin, polarization, energy levels, and spatial modes—presents a novel resource for the enhancement of quantum batteries. Formally, a hyperentangled state resides in a composite Hilbert space, where each constituent subspace corresponds to a distinct DoF of the underlying physical system. Such multi-DoF entanglement enables

the encoding of quantum correlations in a structurally richer tensor product space, with potential applications in improving multiple operational metrics of quantum batteries, including ergotropy, charging power, and scalability. For instance, the presence of multiple DoFs permits concurrent energy transitions in distinct subspaces, increasing the ergotropy density. We can have additive contributions from each DoF, valid under decoupled or weakly coupled interactions, leading to work extraction being parallelized across DoFs, enhancing performance. Even when we talk of charging power, we can see non-trivial contributions due to higher-dimensional correlations. For hyperentangled systems, we can propose the total average charging power satisfies:  $\langle P \rangle_{\tau, \text{hyper}} = \sum_i \langle P_i \rangle_{\tau}$ , in analogy to parallel computing, where each DoF operates as an independent energy channel. The leveraging of orthogonal DoFs allows energy encoding in multiple channels of the same particle, increasing energy throughput while facilitating compact integration in quantum hardware. In the presence of significant coupling, interference terms may lead to non-trivial corrections or even enhanced superadditivity.

### 2.3 The Promise and challenge of cross-DoF dynamics

A novel and potentially transformative direction in the design of quantum batteries is the exploitation of *cross-degree-of-freedom (cross-DoF) cooperative dynamics*, wherein interactions between orthogonal internal quantum properties—such as spin, polarization, and orbital angular momentum—enable emergent thermodynamic pathways inaccessible in conventional architectures. In a hyperentangled system with composite Hilbert space  $\mathcal{H} = \otimes_{i=1}^N \mathcal{H}_{\alpha}^{(i)}$ , the internal Hamiltonian can support both intra-DoF and cross-DoF couplings of the form:

$$H_{\text{int}} = \sum_{\alpha} g_{\alpha} A_{\alpha}^{(1)} \otimes A_{\alpha}^{(2)} + \sum_{\alpha \neq \beta} g_{\alpha\beta} A_{\alpha}^{(1)} \otimes A_{\beta}^{(2)}$$

where  $A_{\alpha}^{(i)}$  are Hermitian operators acting on the  $\alpha$ -th degree of freedom of subsystem  $i$ , and  $g_{\alpha\beta}$  captures the cross-DoF interaction strength. The inclusion of such cross terms breaks the separability of energy subspaces, leading to coherences that couple energy levels across multiple DoFs, and fundamentally alters the structure of passive states. In this setting, the total ergotropy,

$$W_{\text{max}} = \text{Tr}(H_0 \rho) - \text{Tr}(H_0 \rho_{\text{pass}})$$

can exceed the sum of ergotropies from individually entangled or separable DoF subsystems:

$$W_{\text{max}} > \sum_{\alpha} W_{\text{max}}^{(\alpha)}(\rho^{(\alpha)})$$

manifesting a superadditive ergotropy regime. This inequality arises because cross-DoF entanglement introduces global unitary operations capable of harvesting work from inter-DoF coherences, which are inaccessible via local operations. Moreover, the quantum speed limit (QSL) for charging processes is modified by these interactions, as the energy variance  $\Delta E$  is enhanced by non-commuting cross terms. We can then, for a unitary evolution of a pure or mixed state:  $\rho_0 \rightarrow \rho_{\tau}$  under Hamiltonian  $H$ , write a Mandelstam-Tamm-type QSL as

$$\tau_{\text{QSL}} \geq \frac{\hbar \cos^{-1}(F(\rho_0, \rho_\tau))}{\Delta E}$$

where  $F(\rho_0, \rho_\tau) = \text{Tr}[\sqrt{\sqrt{\rho_0}\rho_\tau\sqrt{\rho_0}}]^2$  is the Uhlmann fidelity between the states,  $\Delta E$  is the energy uncertainty (variance) with respect to the generator of the evolution  $H$ , and the cosine inverse term reflects the geometric angle between quantum states. Consequently, we see that the presence of hyperentanglement facilitates both faster charging dynamics and increased work extraction. This framework positions hyperentanglement not simply as a passive quantum resource but as a design principle for engineering many-body systems with optimized energy throughput, particularly in miniaturized quantum energy storage applications where compactness and performance must coexist. It is crucial to reconcile this potential advantage with earlier findings that entanglement is not necessary for optimal work extraction under specific operational paradigms, such as when global unitary operations across the entire system are permitted. The superadditivity enabled by cross-DoF couplings represents a distinct advantage that emerges when operations are constrained by the physical separation of DoFs, or when the goal is not just total work, but maximizing the rate of work extraction (power) within a finite time, as governed by the quantum speed limit. The cross-DoF terms provide a new handle to manipulate the energy variance  $\Delta E$ , directly influencing the charging speed.

### 3 Quantum sensing

Quantum sensors leverage what is often seen as a vulnerability in quantum systems—their extreme sensitivity to external perturbations—as a powerful advantage [37, 38]. This strategic inversion of quantum fragility into functional sensitivity mirrors the early trajectory of semiconductor technologies: long before the digital computer era, devices like selenium-based light meters (for instance, the Weston cell) found widespread commercial use. What is novel today is the increasing focus on engineered quantum systems—often controlled at the level of individual atoms or particles—where entanglement and coherence are intentionally employed as resources to surpass classical limits of measurement sensitivity. In practice, quantum sensing approaches fall into three broad categories. The first involves the use of a quantized physical system, like a spin qubit, a trapped ion, or a superconducting circuit—as a probe of an external quantity [39–41]. The second category extends this idea by exploiting quantum coherence, while the third and most advanced class of quantum sensors employs entanglement as a resource to surpass the sensitivity limits of classical or separable quantum systems. By correlating multiple qubits into a collective quantum state, it becomes possible to achieve measurement precision that scales with the inverse of the number of particles—a regime known as Heisenberg-limited sensing [42–44]. The majority of practical implementations today focus on the first two paradigms due to their scalability and compatibility with existing platforms. These sensors already provide key advantages over classical devices, including high spatial resolution, operation at the nanoscale, and sensitivity to minute changes in external fields.

#### 3.1 Fundamentals of quantum sensing

Any quantum system suitable for sensing applications must exhibit well-defined, resolvable energy levels, often modeled as a two-level system with states  $|0\rangle$  and  $|1\rangle$ , separated

by a transition energy  $\Delta E$ . Within the system, reliable initialization and readout mechanisms are essential, enabling consistent preparation of the sensor and extraction of measurement outcomes. The system should be coherently controllable, typically via time-dependent external fields. Lastly, the quantum sensor must undertake an interaction with an external physical quantity  $V(t)$ , with this interaction quantified by a coupling parameter of the form  $\gamma = \partial^q E / \partial V^q$  with  $q = 1$  or  $q = 2$  for linear or quadratic coupling, respectively. This interaction may induce measurable shifts in the transition energy or drive population changes between quantum states, serving as the basis for sensing.

There are key performance metrics for any quantum sensor, with the two most prominent ones being selectivity and sensitivity [45–48]. A good quantum sensor should respond strongly to the target signal while remaining robust against noise and environmental decoherence. The sensor's intrinsic sensitivity is inversely proportional to  $\gamma\sqrt{T_\chi}$ , where  $\gamma$  is coupling or transduction parameter and  $T_\chi$  is time taken by the system to decohere or relax. For optimization of sensitivity, the coupling parameter should be large and the time taken to decohere should be long. In general, we can model the Hamiltonian for a quantum sensing system as  $H(t) = H_0 + H_V(t) + H_C(t)$ , where we have the internal Hamiltonian  $H_0$  that characterizes the intrinsic properties of the quantum sensor, the signal-dependent Hamiltonian  $H_V(t)$  and the control Hamiltonian  $H_C(t)$ . The term  $H_V(t) = \gamma \vec{V}(t) \cdot \vec{\sigma}$ , where  $\vec{\sigma} = \{\sigma_x, \sigma_y, \sigma_z\}$  are the Pauli matrices, encodes the influence of the external signal  $V(t)$  that the sensor aims to detect. Meanwhile, the control Hamiltonian is associated with externally applied, time-dependent manipulations used to prepare, evolve, or read out the sensor's quantum state. In this context, the central objective of a quantum sensing protocol is to extract information about the unknown signal  $V(t)$  by observing how it perturbs the sensor's dynamics. This is typically achieved through a carefully engineered control strategy  $H_C(t)$ , which is designed to amplify the sensor's response to  $V(t)$  or to suppress decoherence and noise, thereby enhancing sensitivity. Most quantum sensing experiments follow a well-defined sequence consisting of four essential stages: *initialization*, *signal interaction*, *readout*, and *signal estimation*. The process begins by preparing the quantum sensor in a known reference state, often denoted as  $|0\rangle$ . This state is then transformed into a tailored initial sensing state

$$|\psi_0\rangle = U_a|0\rangle$$

typically a superposition of energy eigenstates, through the application of a set of precisely timed control pulses. These operations are represented by the unitary propagator  $U_a$ , which is designed to place the sensor in a state most sensitive to the external signal of interest. Following initialization, the sensor evolves under the influence of a total Hamiltonian  $H(t)$ , which includes contributions from the system's internal dynamics, the external signal, and any applied control fields. This evolution proceeds for a designated sensing duration  $t$ , resulting in a final quantum state

$$|\psi(t)\rangle = U_H(t)|\psi_0\rangle = c_0|0\rangle + c_1|1\rangle,$$

where  $U_H(t)$  is the time-evolution operator generated by  $H(t)$ , and  $|1\rangle$  is a state orthogonal to  $|0\rangle$ . The complex coefficients  $c_0$  and  $c_1$  encode the effect of the signal on the quantum system during the sensing interval. After this evolution, a second unitary

operation  $U_b$  is typically applied to rotate the sensor into a measurement basis that optimally reveals the signal-induced transformation. In many protocols, the initialization and measurement bases are chosen to be the same, though this is not a strict requirement. Under these assumptions, the measured probabilities correspond to the overlaps between the initial and final states. The sensor is then subjected to a projective measurement, yielding a binary outcome corresponding to either  $|0\rangle$  or  $|1\rangle$ . The probability of measuring the excited state  $|1\rangle$  is given by  $p = |c_1|^2$ , which represents the transition probability induced by the signal. This probabilistic outcome is then converted into a classical observable—such as a voltage pulse, photon count, or current—by the measurement apparatus. This measured output provides information about the strength or nature of the external field  $V(t)$  that perturbed the quantum sensor during the interaction time.

These steps comprise a measurement cycle that must be repeated and averaged to determine the transition probability  $p$ , since each measurement yields a binary outcome (0 or 1). Repetitions can be performed sequentially on a single quantum system or in parallel using an ensemble of  $N$  identical, non-interacting systems. Each run yields an estimate of  $p$ , representing the probability of state transition due to the applied signal. While a single value of  $p$  may suffice in some cases, it is often necessary to collect a series  $\{p_k\}$  as a function of evolution time  $t$  or control parameters. Appropriate estimation techniques applied to this dataset enable robust inference of the external signal  $V$ . Uncertainty in  $p$  arises primarily from quantum projection noise, inherent in the stochastic nature of projective measurements [49]. Since each individual trial yields either 0 or 1, high-precision estimation of  $p$  requires averaging over many trials, particularly near  $p = 0.5$  where variance is maximal. Additional sources of error include decoherence and relaxation processes, which degrade the quantum state over time, limiting the effective sensing duration. Imperfections in initialization and control pulses also reduce measurement contrast. Classical readout noise—originating from the conversion of quantum outcomes to classical observables (for instance, voltages, currents, or photon counts)—can further obscure the signal, especially in low-efficiency detection regimes [50, 51]. Depending on noise levels, experiments may operate in single-shot or averaged regimes. The combined quantum and classical uncertainties define the total measurement error, which constrains both the signal-to-noise ratio (SNR) and the sensor's sensitivity. A readout efficiency parameter is often introduced to quantify the deviation from ideal measurement conditions. The SNR improves with increasing measurement repetitions, coherence time, and detection efficiency. For time-dependent signals, stability and noise characteristics can be evaluated using Allan variance [52]. Fundamentally, the Quantum Cramér–Rao Bound (QCRB), governed by the quantum Fisher information, sets the lower bound on estimation variance and defines the ultimate sensitivity limit of quantum sensors, extendable to dynamic signals via quantum spectral estimation methods [53, 54].

### 3.2 Entanglement-enhanced sensing

In conventional quantum sensing, multiple identical and non-interacting quantum probes can be operated in parallel to improve measurement sensitivity. This is commonly realized in ensemble-based sensors such as atomic vapor magnetometers and spin ensembles. Operating  $M$  qubits simultaneously reduces the measurement uncertainty

by a factor of  $1/\sqrt{M}$  due to averaging over quantum projection noise. This defines the *Standard Quantum Limit* (SQL), which mirrors the performance of  $M$  classical sensors running in parallel. However, increasing the number of sensors within a finite volume introduces inter-qubit interactions that may degrade performance, so practical limits are often set by qubit density rather than absolute number. To surpass the SQL and approach the more fundamental *Heisenberg Limit* (HL), where sensitivity scales as  $1/M$ , quantum sensors can be prepared in entangled states. A prominent example is the Greenberger–Horne–Zeilinger (GHZ) state, defined as  $|\psi_0\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes M} + |1\rangle^{\otimes M})$ . This state allows for collective phase accumulation: when exposed to a signal-induced Hamiltonian  $H = \omega_0 \sum_i Z_i$  for time  $t$ , the evolved state acquires a phase  $\phi = M\omega_0 t$ , yielding  $|\psi(t)\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes M} + e^{iM\omega_0 t}|1\rangle^{\otimes M})$ . This  $M$ -fold phase enhancement translates into either shorter interrogation times or improved sensitivity. Critically, the quantum Fisher information (QFI) scales quadratically with  $M$  for GHZ states, enabling Heisenberg-limited estimation  $\Delta\phi \geq 1/M$ , in contrast to the SQL scaling  $\Delta\phi \geq 1/\sqrt{M}$  for unentangled probes.

The sensing protocol with entanglement modifies key steps, resilience and performance [55]. Initialization involves preparing an entangled GHZ state rather than product states. The signal acquisition step accumulates a collective phase across the entangled system. A disentangling operation may then map the global phase into a measurable observable, often requiring only a single qubit measurement due to the symmetry of the GHZ state. The measurement yields an interference fringe pattern with frequency amplified by  $M$ , expressed in the transition probability  $p(t) = \sin^2(M\omega_0 t/2)$ . Despite this theoretical advantage, GHZ states are highly susceptible to decoherence. In the presence of uncorrelated dephasing noise, the decoherence rate of the GHZ state increases linearly with  $M$ , requiring a proportionally shorter interrogation time. This scaling often cancels the potential gain in sensitivity, especially in frequency estimation tasks. Consequently, there is significant interest in alternative entangled states that offer improved robustness. Spin-squeezed states, which reduce quantum fluctuations in one spin component at the cost of increased noise in the conjugate variable, offer sub-SQL scaling while tolerating noise better than GHZ states. Similarly, NOON states—superpositions of  $N$  particles all in one mode or another—have been proposed for enhanced interferometry but suffer from similar fragility. To exploit entangled states in practice, readout strategies like parity detection have been developed. Parity measurements determine whether the number of excitations is even or odd and have been successfully implemented in ion traps and proposed for photonic systems using single-photon detectors or quantum non-demolition techniques. Recent advances in spatially resolved detection and cavity-QED readout have extended parity measurement capabilities to mesoscopic atomic ensembles. While entanglement provides a route to surpass the SQL and achieve Heisenberg-limited sensitivity, the practical benefits depend critically on mitigating decoherence and implementing high-fidelity entangling operations and readouts. This balance often favors the use of partially entangled or spin-squeezed states over maximally entangled GHZ or NOON states in realistic sensing applications.

### 3.3 Hyperentanglement and hybrid entanglement for metrological gain

In quantum sensing, entanglement is a key resource enabling sensitivities beyond the classical limit, particularly through states such as GHZ and spin-squeezed states

which can reach the Heisenberg limit. However, the introduction of *hyperentanglement*—simultaneous entanglement across multiple degrees of freedom (DoFs), such as spin, polarization, orbital angular momentum (OAM), and spatial mode—and *hybrid entanglement*—entanglement between different physical systems or continuous-discrete variable domains—enhances the sensing capabilities significantly. Consider a system of  $M$  particles, each entangled in  $D$  orthogonal DoFs. The total Hilbert space is  $\mathcal{H}_{\text{total}} = \bigotimes_{i=1}^M \left( \bigotimes_{\alpha=1}^D \mathcal{H}_{\alpha}^{(i)} \right)$ , where  $\mathcal{H}_{\alpha}^{(i)}$  is the Hilbert space associated with the  $\alpha$ -th DoF of the  $i$ -th particle. The total quantum state may be written as  $|\Psi\rangle = \sum_{j_1, \dots, j_D} c_{j_1 \dots j_D} |j_1\rangle_{\alpha=1} \otimes \dots \otimes |j_D\rangle_{\alpha=D}$ , encoding entanglement both within individual DoFs and across DoFs simultaneously. The sensing Hamiltonian for an external signal  $\vec{V}(t)$  will have a signal-dependent part of the form

$$H_V(t) = \sum_{\alpha=1}^D \gamma_{\alpha} \vec{V}_{\alpha}(t) \cdot \vec{\sigma}_{\alpha},$$

with  $\vec{\sigma}_{\alpha}$  denoting Pauli matrices acting on DoF  $\alpha$  and  $\gamma_{\alpha}$  the associated coupling strength. The system can also include cross-DoF couplings:  $H_{\text{int}}^{\text{cross}} = \sum_{\alpha \neq \beta} g_{\alpha\beta} \sigma_{\alpha}^{(i)} \otimes \sigma_{\beta}^{(j)}$ , where  $g_{\alpha\beta}$  quantifies the strength of interaction between DoFs  $\alpha$  and  $\beta$ . From a quantum metrology perspective, the sensitivity is bounded by the Quantum Cramér-Rao Bound (QCRB):

$$\Delta^2 \theta \geq \frac{1}{F_Q[\rho_{\theta}]},$$

where  $F_Q[\rho_{\theta}]$  is the quantum Fisher information (QFI). For unitary evolution under  $H = \theta G$ , the QFI is  $F_Q = 4(\Delta G)_{\rho}^2$ . In hyperentangled systems, the generator  $G$  includes both intra- and inter-DoF contributions:  $G = \sum_{\alpha} G_{\alpha} + \sum_{\alpha \neq \beta} G_{\alpha\beta}$ , which may lead to a superadditive QFI:  $F_Q^{\text{hyper}} > \sum_{\alpha} F_Q^{(\alpha)}$ . Consider a hyperentangled GHZ-like state:  $|\Psi_{\text{GHZ}}^{\text{hyper}}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_1^{\otimes M} \dots |0\rangle_D^{\otimes M} + |1\rangle_1^{\otimes M} \dots |1\rangle_D^{\otimes M})$ . Upon evolution, such a state accumulates a phase  $\phi = MD\omega_0 t$ , leading to a transition probability:

$$p(t) = \sin^2 \left( \frac{MD\omega_0 t}{2} \right).$$

This demonstrates an effective enhancement of the metrological resolution scaling with the total “entanglement volume”  $M \times D$ . In *hybrid entangled systems*, different physical subsystems (e.g., light and matter, discrete and continuous variables) are entangled. The total generator of the sensing evolution may take the form:  $G_{\text{hyb}} = G_{\text{disc}} \otimes \mathbb{I}_{\text{cont}} + \mathbb{I}_{\text{disc}} \otimes G_{\text{cont}} + G_{\text{int}}$ , allowing signal acquisition in one modality (e.g., high-resolution continuous variables) and readout in another (e.g., robust discrete qubits). This cross-modality architecture enhances dynamic range, fidelity, and flexibility in practical sensing implementations. In this way, we can see that simultaneous entanglement within and across degrees of freedom—embodied in hyperentangled and hybrid-entangled quantum sensors—provides significant advantages: multi-parameter sensing, superadditive precision scaling, and robustness to decoherence. These entanglement structures extend the fundamental capabilities of quantum sensors beyond the limitations imposed by conventional entanglement.

The idealized phase gain of  $MD$  must be critically examined in the presence of noise. Adding more DoFs ( $D$ ) does not come for free; it potentially introduces new, independent decoherence channels. For example, in a photonic system, entangling polarization and orbital angular momentum (OAM) subjects the state to decoherence in both DoFs, such as polarization-mode dispersion and turbulence-induced OAM scattering. A full metrological analysis would need to consider a noise model where each DoF  $\alpha$  has a characteristic dephasing time  $T_{2,\alpha}$ . The resulting quantum Fisher information would then depend on a competition: the enhanced signal scaling  $\propto (MD)^2$  versus an exponentially decaying coherence term  $\propto \exp[-t(\sum_{\alpha} \Gamma_{\alpha})]$ , where  $\Gamma_{\alpha}$  is the noise rate for DoF  $\alpha$ . The superadditive QFI condition  $\mathcal{F}_Q^{\text{hyper}} > \sum_{\alpha} \mathcal{F}_Q^{(\alpha)}$  is therefore not automatic; it holds only when the coupling to the signal ( $\gamma_{\alpha}$ ) is strong enough and the additional noise from multiplexing DoFs is sufficiently mitigated, for instance, through dynamical decoupling techniques applied in parallel across all DoFs.

## 4 Discussion

The theoretical framework presented above outlines a compelling vision for hyperentanglement-enhanced quantum technologies. Realizing this vision, however, requires confronting significant experimental challenges. This discussion moves from abstract advantage to concrete feasibility, outlining potential platforms, analyzing key hurdles, and proposing a path forward.

### 4.1 Experimental platforms and feasibility

Several physical systems are prime candidates for implementing these ideas.

(a) *For Quantum Batteries* A solid-state platform like Nitrogen-Vacancy (NV) centers in diamond offers a promising testbed. Here, the electronic spin of the NV center could serve as one DoF (e.g., for fast charging), while the surrounding  $C^{13}$  nuclear spins in the lattice could constitute another DoF (e.g., for long-term storage). Cross-DoF couplings ( $g_{\alpha\beta}$ ) are naturally present via the hyperfine interaction and can be controlled using microwave and radio-frequency pulses. However, the challenge lies in achieving the necessary strong coupling strengths ( $g_{\alpha\beta}$ ) that are large compared to decoherence rates ( $\Gamma$ ) to observe superadditive effects before the system loses coherence. Initial estimates suggest that while current NV platforms can achieve coherent control, reaching the strong-coupling regime for cross-DoF interactions required for a significant quantum advantage in power remains a formidable materials-engineering challenge.

(b) *For Quantum Sensors:* Trapped ions or photonic systems are highly suitable. In trapped ions, multiple ions can be entangled in their internal electronic states (one DoF) and their collective motional modes (another DoF), creating a hyperentangled sensor. A hybrid entangled sensor could be realized by entangling a photon (for high-speed, long-distance transmission) with a trapped ion (for long-lived, stable storage and readout). The advantage in sensing would be the ability to encode information about a magnetic field (coupling to the spin DoF) and an electric field gradient (coupling to the motional DoF) simultaneously in a single, shared quantum state, enabling true multi-parameter estimation at the Heisenberg limit.

#### 4.2 Critical challenges and open questions

Despite the potential, several hurdles must be overcome:

- As highlighted in the sensing section, multiplexing DoFs multiplexes noise. Engineering protective protocols, such as multi-DoF dynamical decoupling or operating in decoherence-free subspaces that span multiple DoFs, is an open and critical area of research.
- Preparing, manipulating, and reading out hyperentangled states requires an exponential increase in control precision. A hyper-CPhase gate, for instance, must simultaneously enact conditional operations on multiple DoFs without introducing crosstalk errors. High-fidelity control is paramount.
- Moving from proof-of-principle demonstrations with few particles and DoFs to useful, scalable devices is non-trivial. The 'entanglement volume' ( $M \times D$ ) cannot be increased arbitrarily before the control and decoherence challenges become insurmountable with current technology.

#### 4.3 Towards sustainability

The sustainability link can be made more concrete by focusing on specific applications. For example, a hyperentangled sensor capable of simultaneous high-sensitivity magnetometry and thermometry at the nanoscale could be used to map the heat dissipation and current flow in next-generation low-power microchips, directly aiding in the design of more energy-efficient electronics. For environmental monitoring, such a sensor could precisely characterize the magnetic and thermal signatures of pollutants or biological processes in situ, providing a more complete picture of an ecosystem's health than any single-parameter sensor could.

### 5 Conclusion

This review has identified hyperentanglement and hybrid entanglement as promising, albeit nascent, resources for developing next-generation quantum batteries and sensors within a sustainability-focused framework. By leveraging entanglement across multiple degrees of freedom, we have formalized the potential operational gains in ergotropy, charging speed, and quantum metrological precision. However, this potential is accompanied by significant experimental challenges related to decoherence, control, and scalability. To move from theoretical promise to practical impact, we propose a three-stage research roadmap. In the short term (proof-of-concept), efforts should focus on experimental demonstration of core principles in well-controlled platforms such as NV centers for batteries and trapped ions or photonic systems for sensing, with immediate goals being to demonstrate the parallel charging mechanism in two DoFs of a single system and to show clear superadditive scaling of the quantum Fisher information in a hyperentangled sensor under a simple, well-characterized noise model. Progressing to the mid-term (robustness and control), the community should develop and test dynamical decoupling and error-correction protocols tailored for multi-DoF systems to extend coherence times to practically useful durations, while simultaneously advancing integrated control systems such as quantum-classical feedback loops for stabilizing these complex states. Culminating in the long-term (application-specific devices), with robust control established, the focus can shift to engineering devices for specific sustainability applications, including hyperentangled quantum battery modules for distributed

energy grids or field-deployable hyperentangled sensors for real-time, multi-parameter environmental monitoring of magnetic and electric fields from geological or biological sources. By pursuing this roadmap, the field can transition hyperentanglement from a theoretical abstraction to a core design principle for the next generation of sustainable quantum technologies.

#### Author contributions

MGM was responsible for the conceptualization of the review, comprehensive literature research, investigation and analysis, drafting, and editing of the manuscript.

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#### Data availability

No datasets were generated or analysed during the current study.

#### Declarations

##### Ethics approval and consent to participate

Not applicable

##### Consent for publication

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##### Competing interests

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