© National Institute of Advanced Studies 2001

Published by National Institute of Advanced Studies Indian Institute of Science Campus Bangalore 560 012

Price:Rs.

#### Copies of this lecture can be ordered from:

The Controller National Institute of Advanced Studies Indian Institute of Science Campus Bangalore 560 012 Phone: 080-3344351 Email: mgp@hamsadvani.serc.iisc.ernet.in

ISBN 81-87663-16-2



Typeset & Printed by Verba Network Services 139, Cozy Apts., 8th Main, 12th Cross Malleswaram, Bangalore 560 003 Tel.: 334 6692

# **MOHAMMAD BAGHERI**

Tehran, Iran

The Influence of Indian

Mathematics and Astronomy

in Iran

NIAS LECTURE L2 - 2001



Dear Colleagues, it is a great pleasure for me to be here in a gathering of Indian friends and to talk about "the influence of Indian mathematics and astronomy in Iran". In all my life I have been interested in Indian culture and fascinated by its richness and diversity. As an Iranian, I have always been enthusiastic about the strong and deep common aspects between my native culture and Indian culture. In this context, amongst the similarities which are revealed in appearance, language, traditions etc., the differences have also been interesting to notice.

The fact that the two countries are geographically close to each other and have, to some extent, similar natural environment and climate, has reinforced the common aspects relating to a common Indo-Aryan origin.

### Mohammad Bagheri

We may trace the cultural links through two channels. (i) Our common Indo-Aryan origin which shows itself in the similarities found between Sanskrit and Pahlavi, the ancient languages of India and Iran. Similar mythological elements and physical similarities also fall in this category. (ii) In the course of history, there have been extensive reciprocal contacts between the two nations. This has caused the introduction of Indian words into the Persian language at different periods. Reciprocally, many Persian, Arabic and Turkish words have found their way into India from Iran.

If we look back into the past, we see several group emigrations from Iran to India. Some researchers believe that Hind or Hindustan is the name given to this land by Iranians who entered India through Punjab, which literally means "five rivers", one such major river being the Indus. Jawaharlal Nehru in his Discovery of India mentions the connercial and cultural links between the Sumerites and Accadites of Mesopotamia and the Mohenjudaru. Iran was midway and excavations in West and South Iran have revealed possible connections between the Elamite civilization (ca. 2400-550 B.C.) and the Indus Valley civilization. The evidence includes some mathematical scripts. Last summer, Prof. B.V. Subbarayappa, who studies the probable connections between the proto-Elamite tablets and the Indus scripts, travelled to Iran, studied some tablets in the Iranian National Museum (Tehran), and gave lectures on this subject in Tehran and Isfahan.

Text of Lecture delivered on 20 January 2000 at the NIAS Associates' Programe.

Another contact goes back to the first century A.D., when a group of Muchs or Zoroastrian sages emigrated from Sistan province in south-castern Iran to India. They took their beliefs and knowledge into India and mixed them with the local culture. In Iran, they are called Mugh-Barahmans and also Sun-worshipper Sistanis. In India, they are known by the Sanskrit title Shak Dipy, where Shak refers to their ethnic origin, equivalent to Sistani, and Dipymeans region or island. We have little information about them. The late Jalal Huma'i, a knowledgeable Iranian scholar, mentions that the ancient Indian scientists Varahamihira (ca. 500 A.D.) and Brahmagupta (ca. 600A.D.) were among their descendants. He also writes that these Surworshipper Sistanis still live in India, keeping their distinct identity and way of life. Maybe they are the same Pahlavas who according to Prof. David Pingree established their own kingdom in North-West India. Mary Boyce, in her book on the history of Zoroastrianism, writes that in the first century A.D. a group of Iranians emigrated to India. They worshipped their Sun-God Mithra in the Indian temples of Surya. The Avestan title Raevant, related to the Sanskrit Revant, meaning rich, powerful, or glorious, was used for Mithra, and the same name was thence used for the Indian horse-rider God Revanta.

Varahamihira wrote an astronomical book entitled Brhatsanhita. M. Ramakrishna Bhat who published an English translation and commentary of this work, mentions that

## Mohammad Bagheri

Varahamihira was born near Ujjain. Ramakrishna Bhat believes that Much-Barahmans were people of Indian origin, who had oppre to Iran at an earlier time, and who later returned to India. Varahamihira's Brhatsanhita includes, besides Indian astronomy, materials from Greek and Epyptian astronomy, which may have been introduced through Iran. Brhatsanhita was translated into Persian in the 13th century A.D. Several manuscripts of this Persian translation are kept at Alioparh, London (India Office), Hyderabad, and Lahore. The work has also been translated into Cernan and English. Its English translation has been published at least three times. Al-Binuni has translated Varahamihira's Lachurataka into Arabic. I wish to collect new information about these Much-Barahmans during my present trip to India, and would appreciate any comment or suggestion in this regard. Perhaps further evidence in future will remove the present ambiguities and throw some light on the topic, which in any case refers to cultural exchange between Iran and India.

Later, when the Arabs conquered Iran in the 7th century A.D., a group of Iranians who did not convert to Islam and kept their Zoroastrian beliefs emigrated to India. They are well-known here as Parsis. There is a Persian poem about the adventures of this emigration and the first contacts with the inhabitants of western India. The poem is entitled *The Story of Sanjan*. It was composed by a Zoroastrian priest named Bahram Keyqobad from Navasari in 1599 A.D., and has been translated into Gujarati and English.

Other extensive emigrations into India took place after the invasion of the Mongols in the 13th century and that of the Timurids in the 14th century A.D. Finally, in the 15th and 16th centuries A.D., when scholars, thinkers, and men of literature could not tolerate the difficult circumstances in Iran under the Safavid dynasty, they preferred to leave their country and to find shelter in India, where they could live in comfort and be active. In this period, an Iranian poet, Mulla 'Abd al-Razzaq Lahiji, whose nickname was Fayyaz, wrote a Persian poem about India which may be translated into English as follows:

How nice is India, the Ka'ba of desires! Especially for those friends who look for prosperity, It is incumbent upon rich people to go to Mecca; And those rich in science and art should go to India.

These emigrations caused the propagation of the Persian language in India, mainly after the 10th century A.D. Many Persian books have been composed by Iranians or Indians in India. As the latest example, I maymention *Zij-i Bahadurkhani* composed by Ghulam Hossein Junpuri in 1825. This work contains many Indian materials and terms, and was lithoprinted in India in 1858. It was the main source for the traditional calendar-makers of Iran in the last century. Another example is *Zij-i Muhamad Shahi* by Raja Singh Sawai, composed around 1725 A.D. Prof. Raza Ansari of Aligarh Muslim University has worked on these two astronomical books.

## Mohammad Bagheri

After this general introduction, I want to mention concrete examples of Indian influence on Iranian mathematics and astronomy. The only astronomical work we know from pre-Islamic Iran is Zij-i Shah or Zij-i Shahriyaran, both meaning "The Royal Astronomical Tables". As far as we know, it was composed in the time of the Sassanide Bahram Gur (ca. 450 A.D.). Later, another Sassanide king, Khusro Anushirvan (531-579 A.D.), asked his astronomers to compare Almagest (the famous Greek work by the 2nd century astronomer Ptolemy) and the Indian astronomical tables called Arkanda. They found that the Indian astronomical methods were more desirable. So, they wrote a new version of Zij-i Shah based on Arkanda. No copy of Zij-i Shah has survived. However, fragments of it are saved in the works of Islamic-period Iranian astronomers like al-Biruni, AbuMa'shar, Al-Khwarizmi and Sijzi.

Prof. E. S. Kennedy has shown that *Zij-i Shah* contained parameters coming from the old *Suryasiddhanta*. *Zij-i Shah* also contained Indian methods and parameters, at least with regard to planetary equations. Using the Indian method of 12-digit gnomons for shadow problems was another Indian element in the *Zij-i Shah*. Other elements taken from Indian astronomy were related to the equation of time, lunar crescent visibility, solar and lunar eclipses and parallax.

A series of horoscopes of the vernal equinoxes from *Zij-i Shah* are preserved by Sijzi (a 10th century Iranian mathematician and astronomer from Sistan province who lived in Shiraz). Analysis shows that they were computed by the methods of the *Ardharatrika* or midnight system of Aryabhata.

Through *Zij-i Stah* and the Arabic translations of similar Indian works, Indian astronomy influenced Islamic period astronomy, and reached Spain when it was under Arab domination.

Zij-i Shah existed until the late 9th century A.D., as seen in an epistle of Manuchehr, a Zoroastrian priest (ca. 880 A.D.), to his brother on the subject of ablution, a ceremony to be undergone by a person coming into contact with the dead. There are three letters which have been translated into English, first by E. W. West and later by M. F. Kanga. In the second letter, Manuchehr says that in order to determine the proper time according to the positions of the celestial bodies, astronomers use *zijes* (astronomical tables). He then mentions *Zik-i Shahriyaran, Zik-i Hinduk* (Indian tables) and *Zik-i Ptalanagus* (Ptolemy's *Almagest*).

Ibn al-Nadim (ca. 990) quotes Abu Sahl ibn Naubakht, who worked in the library of Caliph Harun al-Rashid (786-809 A.D.), as saying that in the time of the Sassanide kings Ardeshir and Shapur (3rd century A.D.), the scientific books which were sent to India and China in order to save them

## Mohammad Bagheri

from Alexander's raid, anticipated by the prophet Zoroastre, were again copied and brought to Iran and a certain Indian Frmasp (?) was among those who explained and taught them.

In *Dinkard*, a Pahlavi text of the 9th century A.D., we read that the Indian and Greek works translated into Pahlavi in the time of Ardeshir I and Shapur I (3 rd century A.D.) were revised under Khusro Anushirvan (6th century).

Zij Sindhind is an Arabic translation (ca. 770 A.D.) of one of the Sanskrit Siddhantas, probably the Brahmasiddhanta of Brahmagupta, brought by an Indian astronomer named Kanakah into the court of the Caliph Mansur in Baghdad. This zij exerted a deep influence on the astronomy of the Islamic period, especially in Spain, even after the introduction of Almagest. Kennedy mentions 11 zijes of the Islamic period computed by the methods of Sindhind or strongly affected by them. Among this group of zijes only al-Khwarizmi's has come down to us, not in its original Arabic version but in a 12th century A.D. Latin translation made by Adelard of Bath in Spain, from the redaction made by Maslama al-Majriti (fl. about 1000 A.D.). This zij is influenced by Indian astronomy, directly or through Zij-i Shah.

Kushyar ibn Labban (fl. about 1000 A.D.) was an eminent mathematician from Gilan province of Iran on the southern coast of the Caspian Sea. I was attracted to the history of

mathematics and astronomy by studying his works, and am now preparing an edition of his Arabic *Zij-i Jami'* ("The Comprehensive Astronomical Tables") with an English translation and commentary. He also composed another *zij* entitled *Zij-I Baligh* ("The Extensive Astronomical Tables"). No manuscript of the integral text of *Zij-i Baligh* has been reported up to now. However, a short chapter of this *zij* "On the application of the planets' cycles according to the Indian method" exists in the codex no.86, Mulla Firuz collection, Cama Oriental Institute (Bonbay). I have introduced it in an article presented at the 17th Annual Conference for the History of Arabic Science, Suweida (Syria), 1993.

A major example of the influence of Indian mathematics and astronomy is found in the works of al-Biruni (b. 973 A.D.). He travelled into India in the company of the Ghaznavid king Mahnud. Al-Biruni learned Sanskrit and became familiar with Indian culture and science. He seems to have translated some works from Sanskrit into Arabic and vice versa.

His famous work *India* has been translated from Arabic into English and (partially) into Persian. The work contains ample data about Indian astronomy and astrology, calendars and numeral system. At the end of chapter 26 of *India*, al-Biruni quotes from Brahmagupta's *Brahmasiddhanta* that "the followers of Aryabhata maintain that the earth is moving and the heaven resting. People have tried to refute them by

# Mohammad Bagheri

saying that, if such were the case, stones and trees would fall from the earth". Al-Biruni adds: "But Brahmagupta does not agree with them and says that that would not follow from their theory, apparently because he thought that all heavy things are attracted towards the centre of the earth". Al-Biruni implicitly supports this idea and says: "The rotation of the earth does in no way impair the value of astronomy, as all appearances of an astronomical character can quite as well be explained according to this theory as to the other." But al-Biruni adds: "There are, however, other reasons which make it impossible. This question is most difficult to solve." Sijzi, the Iranian scholar whom I mentioned before, had devised a certain type of astrolabe (an important astronomical device that was the ancestor of present day planispheres) in which the plate representing the stars is fixed and that representing the horizon rotates. Al-Biruni has admired this invention in his book on the astrolabe, and again mentions that some people believe that the apparent daily motion of the stars is due to the rotation of the earth and sky. He adds that the astronomical methods based on both hypotheses may lead to correct calculations and results, and the final judgment about the rotation of earth or sky may only be made by natural philosophers because it is beyond the domain of work of the scientists.

Al-Biruni's *Elements of Astrology*, which he wrote both in Arabic and Persian, contains Indian materials and terms.

Huma'i, the editor of the Persian version, gives a list of 100 Sanskrit names and scientific terms in this work. For example *bhukti*, the daily motion of a celestial body, has been Arabicized as *buht*.

I should also mention here the translation into Arabic of Indian astronomical works like *Aharagana*, *Arkanda* and its source work *Khandakhadyaka*, *Karana tilaka*, *Karana Sara*, and finally the composition, around 1770 in India, of a Persian treatise on modern astronomy by Abu Talib ibn Hasan Husseini Safavi. This treatise is known as the earliest Persian work on modern astronomy.

Now, let us switch to mathematics. One of the most important examples of the Indian influence on Iranian mathematics is the place-value decimal numeral system and the operations made using it. This brilliant system may be evaluated as one of the most important contributions of Indians to the whole of science of human kind. The passing of centuries has seen many theories and methods superseded, while the Indian numeral system has continued to be used universally all over the world.

Before the introduction of the Indian numeral system into Iran, Iranians used a certain type of ciphered numeral system. This system continued to be used until the end of the 7th century A.D. When Arabs conquered Iran, they changed the

# Mohammad Bagheri

Iranian alphabet into Arabic. For numbers, they used the same Iranian ciphered system. However, instead of the Iranian symbols, they used simplified forms of Arabic words for the numbers. This method of writing numbers was called *Siyaqat* and was used in Iran, the Arab countries, the Ottoman Empire, Egypt, and India.

Although there are scattered references to the earlier introduction of Indian numerals in Iran and Europe, it was mainly introduced into Iranian and Islamic period mathematics in the 8th and 9th centuries. Through Islamic civilization it was transmitted to North Africa and Europe. Fibonacci (13th century A.D. Italian mathematician) in his *Liber Abacci* describes how he was surprised when he learned that, by using only the nine Indian digits, one may write down any large number. He adds that he found it more complete than the Greek method. Ibn al-Nadim (10th century A.D.) has given samples of Indian digits in his book *Al-Fihrist*. These numerals were called Indian numerals by the Islamic period mathematicians, while Europeans called them Arabic numerals, because they learned them from the Arabs. Now they are also called Indo-Arabic numerals.

The oldest known work on Indian reckoning written by an Iranian mathematician is *Addition and Subtraction by Indian Reckoning* by Al-Khwarizmi. It was composed in Arabic in the late 8th or early 9th century A.D. The original Arabic

text no longer remains, but a copy of its Latin translation is extant in Cambridge (England). The translation has been made in the 12th century and the manuscript has been copied in the 13th century A.D. This manuscript is incomplete and contains many errors and gaps. However, it has been published several times and has been translated into English, French and Russian. In 1997, Prof. Menso Folkerts from Munich, who had discovered in New York another manuscript of this work which was perfect and complete, published it in Munich with a German translation and commentary. The appearance of this manuscript may be regarded as the major achievement in the history of mathematics in recent decades.

The oldest existing treatise on Indian reckoning written in Islamic period mathematics is *Chapters on Indian Arithmetic* (in Arabic) by Abu Al-Hasan Uglidisi (early 10th century A.D.). The next one is *Principles of Hindu Reckoning* (in Arabic) by Kushyar ibn Labban whom I mentioned before. His book was translated into Hebrew in the 16th century A.D. In recent decades it has been translated into English and French. In 1978 I published a Persian translation in Iran.

Al-Biruni has written a short Arabic treatise entitled *Rashikat al-Hind* ("The Indian Rashiks"). Two manuscripts of this treatise are extant and its text has been published in Hyderabad in 1948. Here al-Biruni mentions proportions as defined and used in India in his time. *Rash* means

### Mohammad Bagheri

astronomical sign, and *rashik* means astrological house. Since, in a problem implying proportions, we find the unknown from three known quantities, it is called *trirashik*. He also mentions inverse proportion, called *viyasta trirashik*, and composite proportions where the unknown is determined from five, seven, nine or more quantities. Biruni gives their Sanskrit names *pancharashika*, *septarashika*, *novarashika*, etc.

Abu al-Wafa Buzjani, the great Iranian mathematician and astronomer of the 10th century, was evidently influenced by Indian mathematics. In his work on arithmetic we find the only reference to negative numbers in Islamic period mathematics. He uses the Arabic name dayn for them, which means debt. This is possibly inspired by the Sanskrit word *kshaya* for debt. He also quotes a method for finding the diameter of the circle which circumscribes a regular polygon of *n* sides. He mentions it as an Indian method. Prof. R.C. Gupta has shown that a formula quite similar to that, based on the method of undetermined coefficients, was given by Bhaskara I in the 7th century A.D.

Omar Khayyam (1048-1131), the famous Iranian mathematician, astronomer and poet, had a treatise on the extraction of square roots and cube roots, by the Indian methods and proofs of their validity. He adds that he has generalized those methods for the extraction of the roots of higher orders. Since this means that Khayyam had found the

coefficients of the binomial expansion, it is a historically important work. He has mentioned this treatise in his *Algebra*, but no manuscript of it has been found as yet.

Finally, I mention that Bhaskaracharya's *Lilawati* was translated into Persian by Faizi Deccani upon Akbar Shah's order in 1587. This Persian translation was published in Calcutta in 1828. Also in 1634, Bhaskaracharya's *Bijaganit* was translated into Persian by Ata'ullah Roshdi upon Shah Jahan's order. The English translation of this work by E. Strachey was made on the basis of the Persian translation.

Indian mathematics and astronomy also received some influence from Iran and the whole Islamic tradition, especially in astronomy. In 1370 A.D. Mahendra Suri, astronomer at the court of Firoz Toghlaq, published his work *Jantraraja* which introduced Arab and Persian astronomy into the Sanskrit Siddhanta tradition. This flow of astronomical ideas as well as instruments continued up to the 17th century A.D., providing the basic materials for those applying the Ptolemaic astronomical system.

Since, perhaps after a long time, I ammaking new connections between the historians of mathematics and astronomy in India and Iran, I feel that my trip has been very fruitful and will promote future scientific exchanges.

## Mohammad Bagheri

## Bibliography

- M. Bagheri, Khayyam's Scientific Legacy, Ganita Bharati, 20 (1998), 1-4, pp.83-91.
- Abu Rayhan Muhammad al-Biruni, Rashikat al-Hind, *Rasa`il Al-Biruni* (Treatises by al-Biruni), part 4, Hyderabad, 1948.
- Id., *al-Tafhim* (Elements of Astrology), ed. J. Huma'i, Tehran, 1983 (in Persian).
- Id., Alberuni's India, ed. E. C. Sachau, 2nd print, New Delhi, 1992.
- M. Folkerts, Die alteste lateinishche Schrift uber das indische Rechnen nach al-Hwarizmi, with cooperation of P. Kunitzsch, Munich, 1997.
- R.C. Gupta, Abu'l Wafa and his Indian Rule About Regular Polygons, *Ganita Bharati*, 14 (1992), 1-4, pp.57-61.
- Id., Spread and Triumph of Indian Numerals, Indian Journal of History of Science, 18 (1983), 1, pp.23-38.
- Id., Indian Astronomy in West Asia, *Vishweshvaranand Indological Journal*, 20 (1982), pp. 219–236.

Ibnal-Nadim, al-Fihrist, ed. R.Tajaddod, Tehran, 1971 (in Arabic).

- E.S. Kennedy, A Survey of Islamic Astronomical Tables, *Transactions of the American Philosophical Society*, 46 (1956), part 2; repr. 1989.
- Abu al-Hasan Kushyar ibn Labban, *Principles of Hindu Reckoning*, tr. M. Levey and M. Petruck, Wisconsin, 1965.
- Abual-Hasan Ahmad Uqlidisi, *Kitabal-Usul fi al-Hisabal-Hindi* (The Story of Hindu-Arabic Arithmetic), ed. A.S. Saidan, Dordrecht/Boston, 1978.